EMULATED CURRENT MODE CONTROL FOR BUCK REGULATORS USING SAMPLE AND HOLD TECHNIQUE

Small Signal Linear Analysis and Comparison to Peak and Valley Methods

by

Robert Sheehan
Principal Applications Engineer
National Semiconductor Corporation
Santa Clara, CA

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Abstract – While naturally sampled peak and valley current mode control methods have been widely used, other control architectures are possible using gated sampling techniques. Theory for an emulated peak current mode control method using a gated sample and hold of the valley current is developed. This gated sampling technique removes the duty cycle dependence of the slope compensating ramp, stabilizing the modulator gain over changes in line voltage. A general solution for current mode buck regulator small signal linear equations is presented. This allows the modulator gain for any control method to be introduced into the equations, including peak, valley, average and gated sampling methods. Comparison to peak and valley is made using switching, linear and LaPlace spice models. Sub-harmonic stability bounds are demonstrated using graphical spreadsheet calculators. Theory is verified with frequency response measurements of an actual circuit.

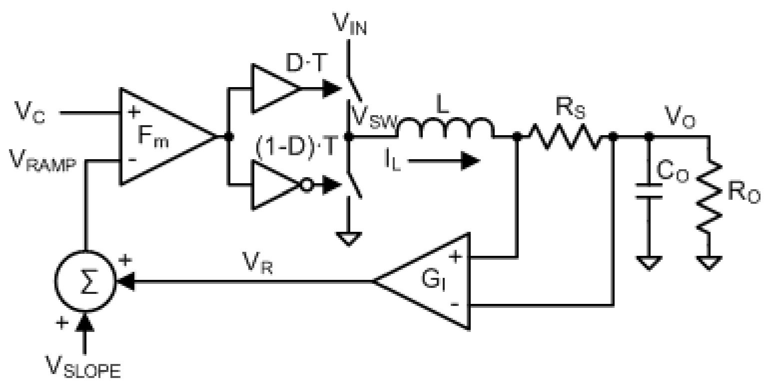

Figure 1: Naturally sampled peak or valley current mode buck regulator.

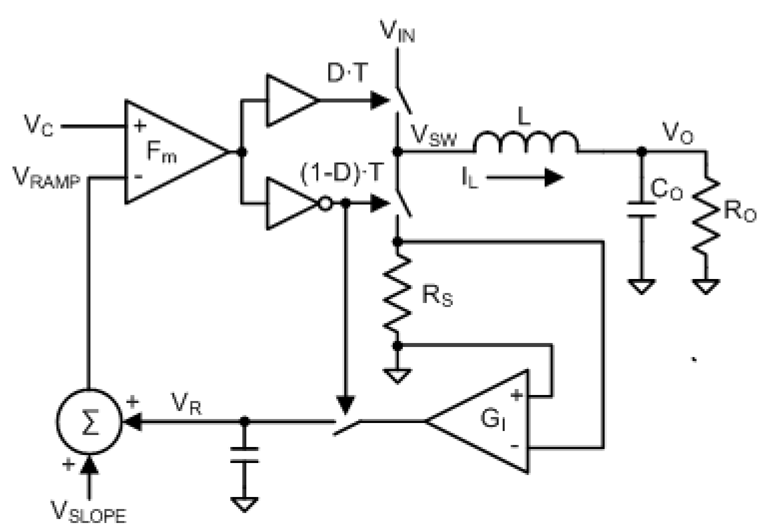

Figure 2: Emulated peak current mode buck regulator using valley sample and hold.

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1. Introduction

There are a lot of misconceptions and misinformation about current mode control in the industry. Papers that have been written at the graduate or PhD level are hard to understand. Concepts are difficult to put into practical use.

Basically, an ideal current mode converter is only dependent on the dc or average inductor current. The inner current loop turns the inductor into a voltage controlled current source, effectively removing the inductor from the outer voltage control loop at dc and low frequencies. The current loop gain splits the complex conjugate pole of the output filter into two real poles, so that the characteristic of the output filter is set by the capacitor and load resistor. Only when the impedance of the output inductor equals the current loop gain does the inductor pole reappear at higher frequencies.

Whether the current mode converter is peak, valley, average, or sample and hold is secondary to the operation of the current loop. As long as the dc current is sampled, current mode operation is maintained. The modulator gain is dependent on the effective slope of the ramp presented to the modulating comparator input. Each operating mode will have a unique characteristic equation for the modulator gain. The requirement for slope compensation is dependent on the relationship of the average current to the value of current at the time when the sample is taken.

To understand the theory and follow the derivations presented in this paper, a good working knowledge of the references is needed. For the practical designer, simplified transfer functions along with tabulated general gain parameters provide the basic tools for design analysis.

The primary application for emulated current mode is high input voltage to low output voltage operating at a narrow duty cycle. In any practical design, device capacitance may cause a significant leading edge spike on the current sense waveform. By sampling the inductor current at the end of the switching cycle and adding an external ramp, the minimum on time can be significantly reduced, without the need for blanking or filtering which is normally required for peak current mode control.

2. Linear Modeling

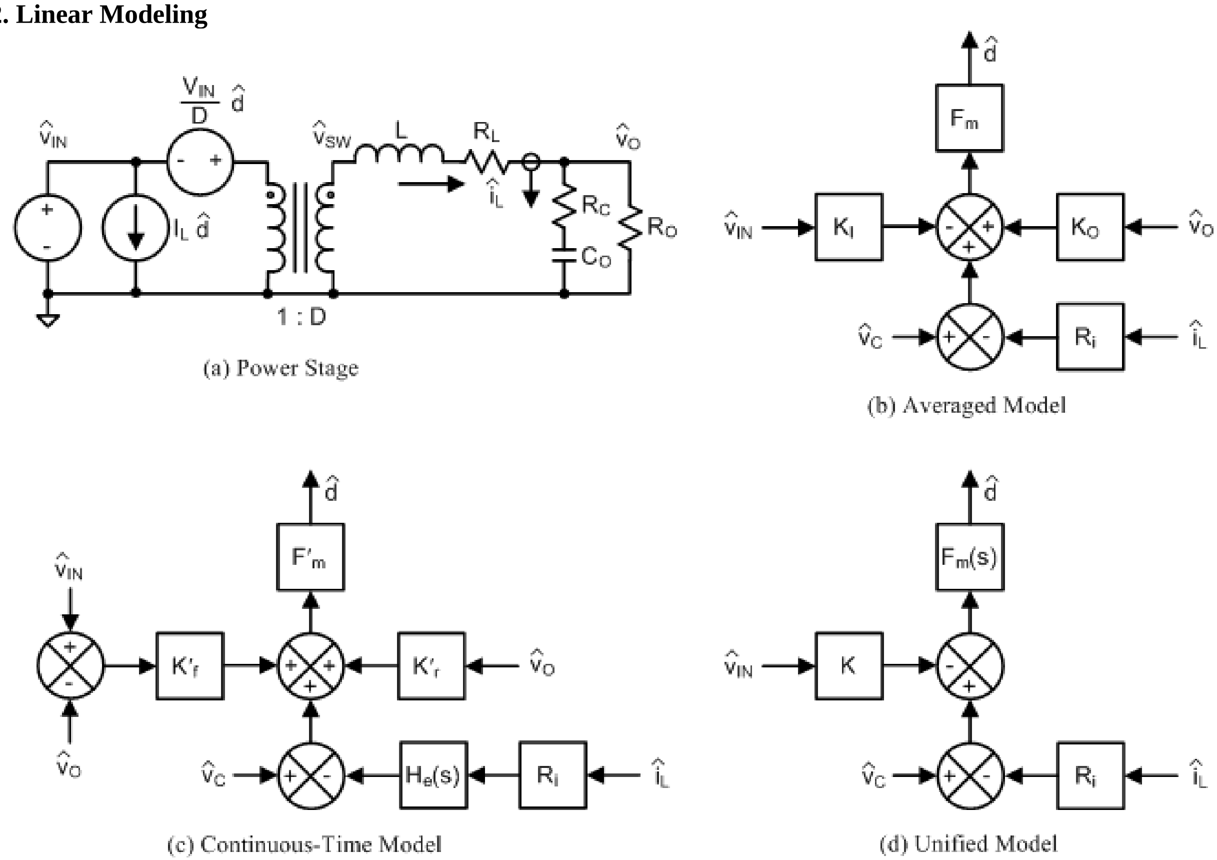


Figure 3: Buck regulator linear models.

Averaged Model

Reference [1] has been one of the most popular papers covering current mode control. The analysis presented here refers the inductor current to the control voltage as the basis for writing the transfer functions.

Starting with $\hat{v}_O$, write the transfer function in terms of voltages:

$$\hat{v}_O = \hat{v}_{SW} \cdot \frac{Z_O}{Z_O + Z_L} \tag{1}$$

$$\hat{v}_{SW} = \hat{v}_{IN} \cdot D + V_{IN} \cdot \hat{d} \tag{2}$$

$$\hat{d} = F_m \cdot (\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K_I + \hat{v}_O \cdot K_O) \tag{3}$$

$$\hat{i}_L = \frac{\hat{v}_O}{Z_O} \tag{4}$$

Combining equations 1 through 4 yields:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + V_{IN} \cdot F_m \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{Z_O} - \hat{v}_{IN} \cdot K_I + \hat{v}_O \cdot K_O\right)\right] \cdot \frac{Z_O}{Z_O + Z_L} \tag{5}$$

Define K_{mp} as:

$$K_{mp} = V_{IN} \cdot F_m \tag{6}$$

Setting $\hat{v}_{IN}$ to zero allows the control to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{K_{mp} \cdot Z_O}{Z_O + Z_L + K_{mp} \cdot (R_i - K_O \cdot Z_O)} \tag{7}$$

Setting $\hat{v}_C$ to zero allows the line to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{(D - K_{mp} \cdot K_I) \cdot Z_O}{Z_O + Z_L + K_{mp} \cdot (R_i - K_O \cdot Z_O)} \tag{8}$$

Define Z_O and Z_L:

$$Z_O = \left(\frac{1}{s \cdot C_O} + R_C\right) \| R_O = \frac{R_O \cdot (1 + s \cdot C_O \cdot R_C)}{1 + s \cdot C_O \cdot (R_O + R_C)} \tag{9}$$

$$Z_L = s \cdot L + R_L + R_S \tag{10}$$

Equations 7 and 8 can be plotted without further analysis using MATHCAD, SPICE or other application. Define terms for:

K_{mp}	Modulator gain coefficient
K_I	Line to modulator gain block
K_O	Output to modulator gain block
R_i	Current sense amplifier gain = $G_I \cdot R_S$

DC Transfer Functions

Control to Output

Let $Z_O=R_O$, $Z_L=R_L+R_S$. From equation 7:

$$\frac{\hat{v}_O}{\hat{v}_C}(dc) = \frac{K_{mp} \cdot R_O}{R_O + R_L + R_S + K_{mp} \cdot (R_i - K_O \cdot R_O)} \tag{11}$$

By factoring this becomes:

$$\frac{\hat{v}_O}{\hat{v}_C}(dc) = \frac{R_O}{R_i} \cdot \frac{1}{1 + \frac{R_L + R_S}{K_{mp} \cdot R_i} + R_O \cdot \frac{1 - K_{mp} \cdot K_O}{K_{mp} \cdot R_i}} \tag{12}$$

Define K_m as the modulator gain where:

$$K_m = \frac{1}{\frac{1}{K_{mp}} - K_O} \tag{13}$$

This allows the control to output gain to be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_C}(dc) = \frac{R_O}{R_i} \cdot \frac{1}{1 + \frac{R_L + R_S}{K_{mp} \cdot R_i} + \frac{R_O}{K_m \cdot R_i}} \tag{14}$$

If $R_O \gg R_L+R_S$ the simplified expression is:

$$\frac{\hat{v}_O}{\hat{v}_C}(dc) \approx \frac{R_O}{R_i} \cdot \frac{1}{1 + \frac{R_O}{K_m \cdot R_i}} \tag{15}$$

Line to Output

In similar fashion:

$$\frac{\hat{v}_O}{\hat{v}_{IN}}(dc) = \frac{(D - K_{mp} \cdot K_I) \cdot R_O}{R_O + R_L + R_S + K_{mp} \cdot (R_i - K_O \cdot R_O)} \tag{16}$$

By factoring this becomes:

$$\frac{\hat{v}_O}{\hat{v}_{IN}}(dc) = \frac{R_O \cdot D}{R_i} \cdot \frac{\frac{1}{K_{mp}} - \frac{K_I}{D}}{1 + \frac{R_L + R_S}{K_{mp} \cdot R_i} + R_O \cdot \frac{1 - K_{mp} \cdot K_O}{K_{mp} \cdot R_i}} \tag{17}$$

Define K_n as the audio susceptibility coefficient:

$$K_n = \frac{1}{K_{mp}} - \frac{K_I}{D} \tag{18}$$

This allows the line to output gain to be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_{IN}}(dc) = \frac{R_O \cdot D}{R_i} \cdot \frac{K_n}{1 + \frac{R_L + R_S}{K_{mp} \cdot R_i} + \frac{R_O}{K_m \cdot R_i}} \quad (19)$$

If $R_O >> R_L + R_S$ the simplified expression is:

$$\frac{\hat{v}_O}{\hat{v}_{IN}}(dc) \approx \frac{R_O \cdot D}{R_i} \cdot \frac{K_n}{1 + \frac{R_O}{K_m \cdot R_i}} \quad (20)$$

Continuous-Time Model

References [2] and [3] cover this model. The sampling gain is incorporated into the current loop as $H_e(s)$. Derivation of the gain blocks for the on voltage and off voltage requires differentiation of the inductor current with respect to each voltage. Here, a straightforward algebraic method is used to relate the continuous-time model to the averaged model. This provides a simple means to derive the equations for different control modes.

Starting with $\hat{d}$, write the transfer function in terms of voltages:

$$\hat{d} = F'_m \cdot (\hat{v}_C - \hat{i}_L \cdot R_i \cdot H_e(s) + \hat{v}_{ON} \cdot K'_f + \hat{v}_{OFF} \cdot K'_r) \quad (21)$$

Combining with equations 1, 2 and 4 for the power stage yields:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + V_{IN} \cdot F'_m \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i \cdot H_e(s)}{Z_O} + \hat{v}_{ON} \cdot K'_f + \hat{v}_{OFF} \cdot K'_r \right) \right] \cdot \frac{Z_O}{Z_O + Z_L} \quad (22)$$

Define K'_{mp} as:

$$K'_{mp} = V_{IN} \cdot F'_m \quad (23)$$

Let $\hat{v}_{ON} = \hat{v}_{IN} - \hat{v}_O$, $\hat{v}_{OFF} = \hat{v}_O$, $Z_O = R_O$, $Z_L = 0$, $H_e(s) = 1$. Solve for the dc gain equation.

$$\hat{v}_O(dc) \approx \left[\hat{v}_{IN} \cdot D + K'_{mp} \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{R_O} + \hat{v}_{IN} \cdot K'_f - \hat{v}_O \cdot K'_f + \hat{v}_O \cdot K'_r \right) \right] \quad (24)$$

Setting $\hat{v}_{IN}$ to zero allows the control to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_C}(dc) \approx \frac{R_O}{R_i} \cdot \frac{1}{1 + R_O \cdot \frac{1 + K'_{mp} \cdot K'_f - K'_{mp} \cdot K'_r}{K'_{mp} \cdot R_i}} \quad (25)$$

Setting $\hat{v}_C$ to zero allows the line to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_{IN}}(dc) \approx \frac{R_O \cdot D}{R_i} \cdot \frac{\frac{1}{K'_{mp}} + \frac{K'_f}{D}}{1 + R_O \cdot \frac{1 + K'_{mp} \cdot K'_f - K'_{mp} \cdot K'_r}{K'_{mp} \cdot R_i}} \quad (26)$$

Define K'_m and K'_n where:

$$K'_m = \frac{1}{\frac{1}{K'_{mp}} + K'_f - K'_r} \qquad K'_n = \frac{1}{K'_{mp}} + \frac{K'_f}{D} \tag{27}$$

Equate K_m to K'_m and K_n to K'_n. Solve for K'_f and K'_r.

$$K'_f = D \cdot \left(\frac{1}{K_{mp}} - \frac{1}{K'_{mp}} \right) - K_I \qquad K'_r = \frac{1}{K'_{mp}} - \frac{1}{K_{mp}} + K'_f + K_O \tag{28}$$

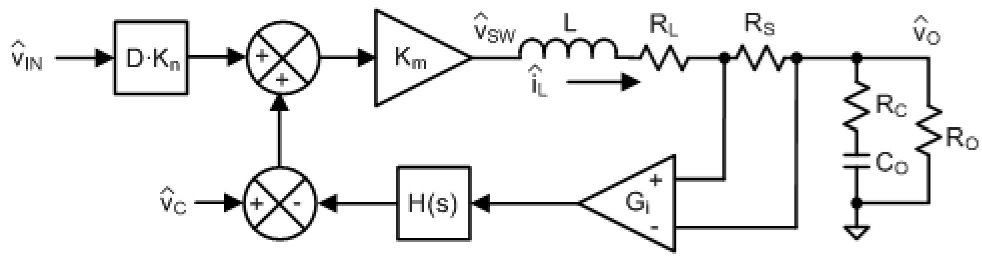


Figure 4: Simplified continuous-time model using general gain parameters. This model is valid for control to output transfer functions of all operating modes. The line to output transfer function is valid at dc, but diverges from the actual response over frequency.

$$H(s) = H_e(s) + \frac{Z_L}{R_i} \cdot \left(\frac{1}{K'_{mp}} - \frac{1}{K_m} \right) \tag{29}$$

Where:

$$H_e(s) = 1 + \frac{s}{\omega_n \cdot Q_z} + \frac{s^2}{\omega_n^{\ 2}} \qquad \omega_n = \frac{\pi}{T} \qquad Q_z = -\frac{2}{\pi} \tag{30}$$

Unified Model

The unified model is presented in references [4] and [5]. This uses a single pole in series with the modulator to account for the sampling gain.

Starting with $\hat{d}$, write the transfer function in terms of voltages:

$$\hat{d} = F_m(s) \cdot (\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K) \tag{31}$$

Combining with equations 1, 2 and 4 for the power stage yields:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + V_{IN} \cdot F_m(s) \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{Z_O} - \hat{v}_{IN} \cdot K \right) \right] \cdot \frac{Z_O}{Z_O + Z_L} \tag{32}$$

Define:

$$V_{IN} \cdot F_m(s) = K_m \cdot H_p(s) \tag{33}$$

Let Z_O=R_O, Z_L=0, $H_p(s)$=1. Solve for the dc gain equation.

$$\hat{v}_O(dc) \approx \left[\hat{v}_{IN} \cdot D + K_m \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{R_O} - \hat{v}_{IN} \cdot K\right)\right] \tag{34}$$

Setting $\hat{v}_{IN}$ to zero allows the control to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_C}(dc) \approx \frac{R_O}{R_i} \cdot \frac{1}{1 + \frac{R_O}{K_m \cdot R_i}} \tag{35}$$

Setting $\hat{v}_C$ to zero allows the line to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_{IN}}(dc) \approx \frac{R_O \cdot D}{R_i} \cdot \frac{\frac{1}{K_m} - \frac{K}{D}}{1 + \frac{R_O}{K_m \cdot R_i}} \tag{36}$$

Comparing this to the averaged and continuous-time models:

$$K_n = \frac{1}{K_m} - \frac{K}{D} \qquad \text{Therefore:} \qquad K = D \cdot \left(\frac{1}{K_m} - K_n\right) \tag{37}$$

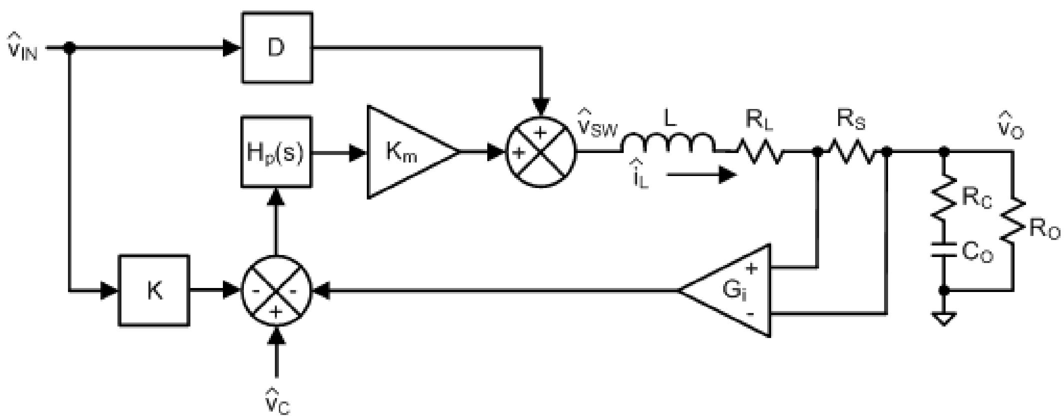


Figure 5: Unified model uses the high frequency asymptote for a single pole in series with the modulator. This accurately models the current loop and control to output transfer functions, but is limited to PCM1, VCM1 and VCM3 operating modes.

$$H_p(s) = \frac{1}{1 + \frac{s}{\omega_{Hp}}} \qquad \text{Where:} \quad \omega_{Hp} = \frac{\omega_n}{Q} \tag{38}$$

Algebraic manipulation of the closed loop continuous-time expression for H(s) to find a general open loop expression for $H_p(s)$ would appear to be straightforward. For peak or valley current mode with a fixed slope compensating ramp, $\frac{s}{\omega_n \cdot Q_z} \cong -\frac{Z_L}{R_i} \cdot \left(\frac{1}{K'_{mp}} - \frac{1}{K_m}\right)$. H(s) reduces to $1 + \frac{s^2}{{\omega_n}^2}$, which leads to the simplification of a single pole for $H_p(s)$. This is not the case for other operating modes. Further work is needed to develop a general expression for $H_p(s)$. Though the unified model shows the potential to accurately model the line to output transfer function, this has not been validated. Comparisons of line to output bode plots from the linear model to SPICE results from the switched model do not match over frequency.

Simplified Transfer Functions

The simplified transfer functions assume poles that are well separated by the current loop gain. Expressions for the averaged model do not show the additional phase shift due to the sampling effect. The control to output gain of the continuous-time model accurately represents the circuit's behavior to half the switching frequency. The line to output expressions for audio susceptibility are accurate at dc, but diverge from the actual response over frequency.

Averaged Model:

$$\frac{\hat{v}_O}{\hat{v}_C} \approx \frac{R_O}{R_i} \cdot \frac{1}{1+\frac{R_O}{K_m \cdot R_i}} \cdot \frac{1+\frac{s}{\omega_Z}}{\left(1+\frac{s}{\omega_P}\right)\cdot\left(1+\frac{s}{\omega_L}\right)} \tag{39}$$

$$\frac{\hat{v}_O}{\hat{v}_{IN}} \approx \frac{R_O \cdot D}{R_i} \cdot \frac{K_n}{1+\frac{R_O}{K_m \cdot R_i}} \cdot \frac{1+\frac{s}{\omega_Z}}{\left(1+\frac{s}{\omega_P}\right)\cdot\left(1+\frac{s}{\omega_L}\right)} \tag{40}$$

Continuous-Time Model:

$$\frac{\hat{v}_O}{\hat{v}_C} \approx \frac{R_O}{R_i} \cdot \frac{1}{1+\frac{R_O}{K_m \cdot R_i}} \cdot \frac{1+\frac{s}{\omega_Z}}{\left(1+\frac{s}{\omega_P}\right)\cdot\left(1+\frac{s}{\omega_n \cdot Q}+\frac{s^2}{\omega_n^{\,2}}\right)} \tag{41}$$

$$\frac{\hat{v}_O}{\hat{v}_{IN}} \approx \frac{R_O \cdot D}{R_i} \cdot \frac{K_n}{1+\frac{R_O}{K_m \cdot R_i}} \cdot \frac{1+\frac{s}{\omega_Z}}{\left(1+\frac{s}{\omega_P}\right)\cdot\left(1+\frac{s}{\omega_n \cdot Q}+\frac{s^2}{\omega_n^{\,2}}\right)} \tag{42}$$

Where: $\omega_Z = \frac{1}{C_O \cdot R_C}$ $\quad \omega_P \approx \frac{1}{C_O}\cdot\left(\frac{1}{R_O}+\frac{1}{K_m \cdot R_i}\right)$ $\quad \omega_L \approx \frac{K_{mp} \cdot R_i}{L}$ $\quad \omega_n = \frac{\pi}{T}$ (43)

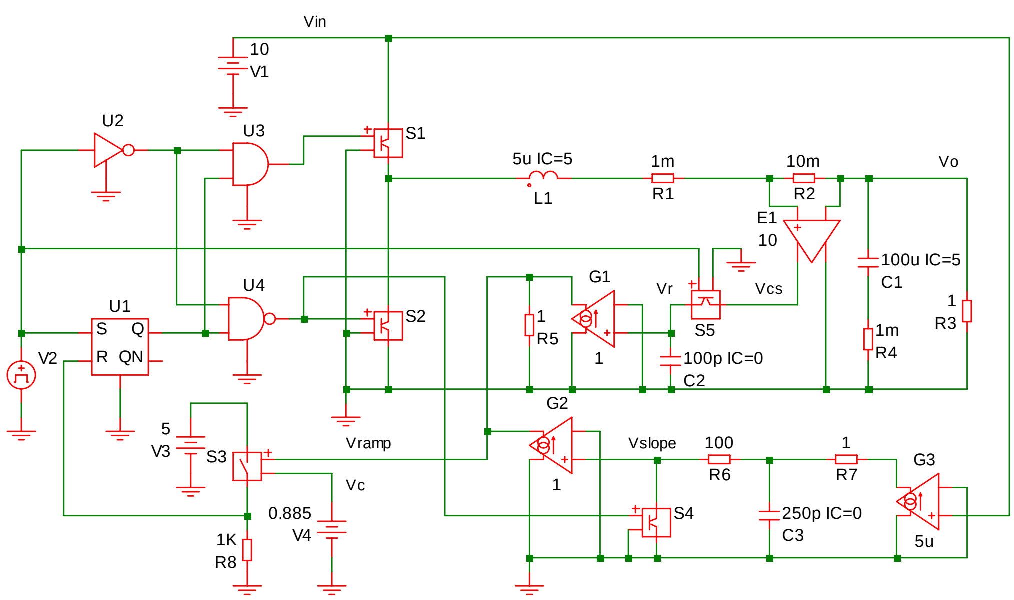


Figure 6: SIMPLIS SPICE schematic - emulated peak current mode synchronous buck.

TABLE 1A

SUMMARY OF GENERAL GAIN PARAMETERS $V_{SLOPE} = S_e \cdot T$ $\omega_n = \frac{\pi}{T}$

Mode	S_e, S_n	m_c, Q	K_m, K_n
PCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$m_C = 1 + \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C \cdot D' - 0.5)}$	$K_m = \frac{1}{(0.5 - D) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}}$ $K_n = \frac{V_{SL}}{V_{IN}} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$
PCM2	$S_e = \frac{V_O \cdot K_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$m_C = 1 + \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C \cdot D' - 0.5)}$	$K_m = \frac{1}{(0.5 - D) \cdot R_i \cdot \frac{T}{L} + 2 \cdot K_{SL} \cdot D}$ $K_n = \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$
VCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$m_C = 1 + \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C \cdot D - 0.5)}$	$K_m = \frac{1}{(D - 0.5) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}}$ $K_n = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D + \frac{V_{SL}}{V_{IN}}$
VCM2	$S_e = \frac{(V_{IN} - V_O) \cdot K_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$m_C = 1 + \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C \cdot D - 0.5)}$	$K_m = \frac{1}{(D - 0.5) \cdot R_i \cdot \frac{T}{L} + 2 \cdot K_{SL} \cdot D'}$ $K_n = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D + \frac{K_{SL}}{D} - K_{SL} \cdot D$
EPCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$m_C = \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C - 0.5)}$	$K_m = \frac{1}{(D - 0.5) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}}$ $K_n = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D + \frac{V_{SL}}{V_{IN}}$
EPCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$m_C = \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C - 0.5)}$	$K_m = \frac{1}{(D - 0.5) \cdot R_i \cdot \frac{T}{L} + K_{SL}}$ $K_n = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$
EVCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$m_C = \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C - 0.5)}$	$K_m = \frac{1}{(0.5 - D) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}}$ $K_n = \frac{V_{SL}}{V_{IN}} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$
EVCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$m_C = \frac{S_e}{S_n}$ $Q = \frac{1}{\pi \cdot (m_C - 0.5)}$	$K_m = \frac{1}{(0.5 - D) \cdot R_i \cdot \frac{T}{L} + K_{SL}}$ $K_n = \frac{K_{SL}}{D} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$

TABLE 1B

SUMMARY OF GENERAL GAIN PARAMETERS $V_{SLOPE} = S_e \cdot T$ $\omega_n = \dfrac{\pi}{T}$

Mode	S_e, S_n	m_c, Q	K_m, K_n
VCM3	$S_e = \dfrac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \dfrac{V_O \cdot R_i}{L}$	$m_C = 1 + \dfrac{S_e}{S_n}$ $Q = \dfrac{1}{\pi \cdot (m_C \cdot D - 0.5)}$	$K_m = \dfrac{1}{(D-0.5) \cdot R_i \cdot \frac{T}{L} + K_{SL}}$ $K_n = 0.5 \cdot R_i \cdot \dfrac{T}{L} \cdot D + \dfrac{K_{SL}}{D}$
EPCM3	$S_e = \dfrac{(V_{IN} - V_O) \cdot K_{SL} + V_{SL}}{T}$ $S_n = \dfrac{V_{IN} \cdot R_i}{L}$	$m_C = \dfrac{S_e}{S_n}$ $Q = \dfrac{1}{\pi \cdot (m_C - 0.5)}$	$K_m = \dfrac{1}{(D-0.5) \cdot R_i \cdot \frac{T}{L} + (1 - 2 \cdot D) \cdot K_{SL} + \frac{V_{SL}}{V_{IN}}}$ $K_n = (0.5 \cdot R_i \cdot \dfrac{T}{L} - K_{SL}) \cdot D + \dfrac{V_{SL}}{V_{IN}}$
EPCM4	$S_e = \dfrac{V_{IN} \cdot K_{SL} + V_{SL}}{T}$ $S_n = \dfrac{V_{IN} \cdot R_i}{L}$	$m_C = \dfrac{S_e}{S_n}$ $Q = \dfrac{1}{\pi \cdot (m_C - 0.5)}$	$K_m = \dfrac{1}{(D-0.5) \cdot R_i \cdot \frac{T}{L} + K_{SL} + \frac{V_{SL}}{V_{IN}}}$ $K_n = 0.5 \cdot R_i \cdot \dfrac{T}{L} \cdot D + \dfrac{V_{SL}}{V_{IN}}$

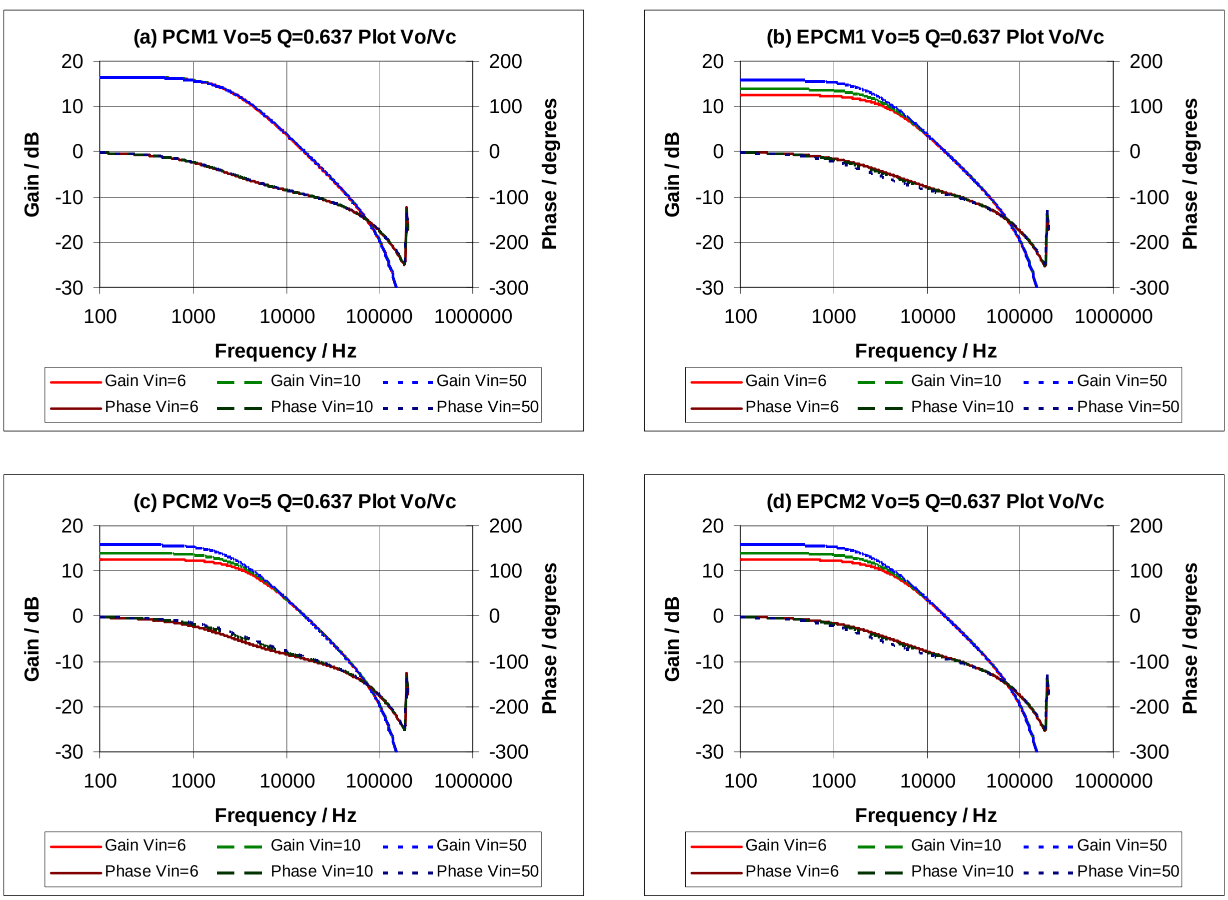


Figure 7: SIMPLIS SPICE results of control to output gain.

TABLE 2A CONTROL VOLTAGE AND DUTY CYCLE EQUATIONS FOR AVERAGED MODEL		
Mode	S_e, S_n	$v_C, \hat{d}$
PCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$i_L \cdot R_i + 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + V_{SL} \cdot d = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right) + \hat{v}_O \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)}{0.5 \cdot (V_{IN} - V_O) \cdot R_i \cdot \frac{T}{L} + V_{SL}}$
PCM2	$S_e = \frac{V_O \cdot K_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$i_L \cdot R_i + 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + v_O \cdot K_{SL} \cdot d = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right) + \hat{v}_O \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} - K_{SL}\right) \cdot D}{0.5 \cdot (V_{IN} - V_O) \cdot R_i \cdot \frac{T}{L} + V_O \cdot K_{SL}}$
VCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$i_L \cdot R_i - 0.5 \cdot v_O \cdot (1-d) \cdot \frac{T}{L} \cdot R_i - V_{SL} \cdot (1-d) = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i + \hat{v}_O \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} \cdot (1-D)\right)}{0.5 \cdot V_O \cdot R_i \cdot \frac{T}{L} + V_{SL}}$
VCM2	$S_e = \frac{(V_{IN} - V_O) \cdot K_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$i_L \cdot R_i - 0.5 \cdot v_O \cdot (1-d) \cdot \frac{T}{L} \cdot R_i - (v_{IN} - v_O) \cdot K_{SL} \cdot (1-d) = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(-K_{SL} \cdot (1-D)\right) + \hat{v}_O \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} - K_{SL}\right) \cdot (1-D)}{0.5 \cdot V_O \cdot R_i \cdot \frac{T}{L} + (V_{IN} - V_O) \cdot K_{SL}}$
EPCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$i_L \cdot R_i - 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + V_{SL} \cdot d = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right) + \hat{v}_O \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)}{0.5 \cdot (V_O - V_{IN}) \cdot R_i \cdot \frac{T}{L} + V_{SL}}$
EPCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$i_L \cdot R_i - 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + v_{IN} \cdot K_{SL} \cdot d = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D + \hat{v}_O \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)}{0.5 \cdot (V_O - V_{IN}) \cdot R_i \cdot \frac{T}{L} + V_{IN} \cdot K_{SL}}$
EVCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$i_L \cdot R_i + 0.5 \cdot v_O \cdot (1-d) \cdot \frac{T}{L} \cdot R_i - V_{SL} (1-d) = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i + \hat{v}_O \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot (1-D)\right)}{V_{SL} - 0.5 \cdot V_O \cdot R_i \cdot \frac{T}{L}}$
EVCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$i_L \cdot R_i + 0.5 \cdot v_O \cdot (1-d) \cdot \frac{T}{L} \cdot R_i - v_{IN} \cdot K_{SL} \cdot (1-d) = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(-K_{SL} \cdot (1-D)\right) + \hat{v}_O \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot (1-D)\right)}{V_{IN} \cdot K_{SL} - 0.5 \cdot V_O \cdot R_i \cdot \frac{T}{L}}$

TABLE 2B
CONTROL VOLTAGE AND DUTY CYCLE EQUATIONS FOR AVERAGED MODEL

Mode	S_e, S_n	$v_C, \hat{d}$
VCM3	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$i_L \cdot R_i - 0.5 \cdot v_O \cdot (1-d) \cdot \frac{T}{L} \cdot R_i - v_{IN} \cdot K_{SL} \cdot (1-d) = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot (-K_{SL} \cdot (1-D)) + \hat{v}_O \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot (1-D)}{0.5 \cdot V_O \cdot R_i \cdot \frac{T}{L} + V_{IN} \cdot K_{SL}}$
EPCM3	$S_e = \frac{(V_{IN} - V_O) \cdot K_{SL} + V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$i_L \cdot R_i - 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + (v_{IN} - v_O) \cdot K_{SL} \cdot d + V_{SL} \cdot d = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D + \hat{v}_O \cdot \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D}{(V_{IN} - V_O) \cdot (K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}) + V_{SL}}$
EPCM4	$S_e = \frac{V_{IN} \cdot K_{SL} + V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$i_L \cdot R_i - 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + v_{IN} \cdot K_{SL} \cdot d + V_{SL} \cdot d = v_C$ $\hat{d} = \frac{\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D + \hat{v}_O \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)}{0.5 \cdot (V_O - V_{IN}) \cdot R_i \cdot \frac{T}{L} + V_{IN} \cdot K_{SL} + V_{SL}}$

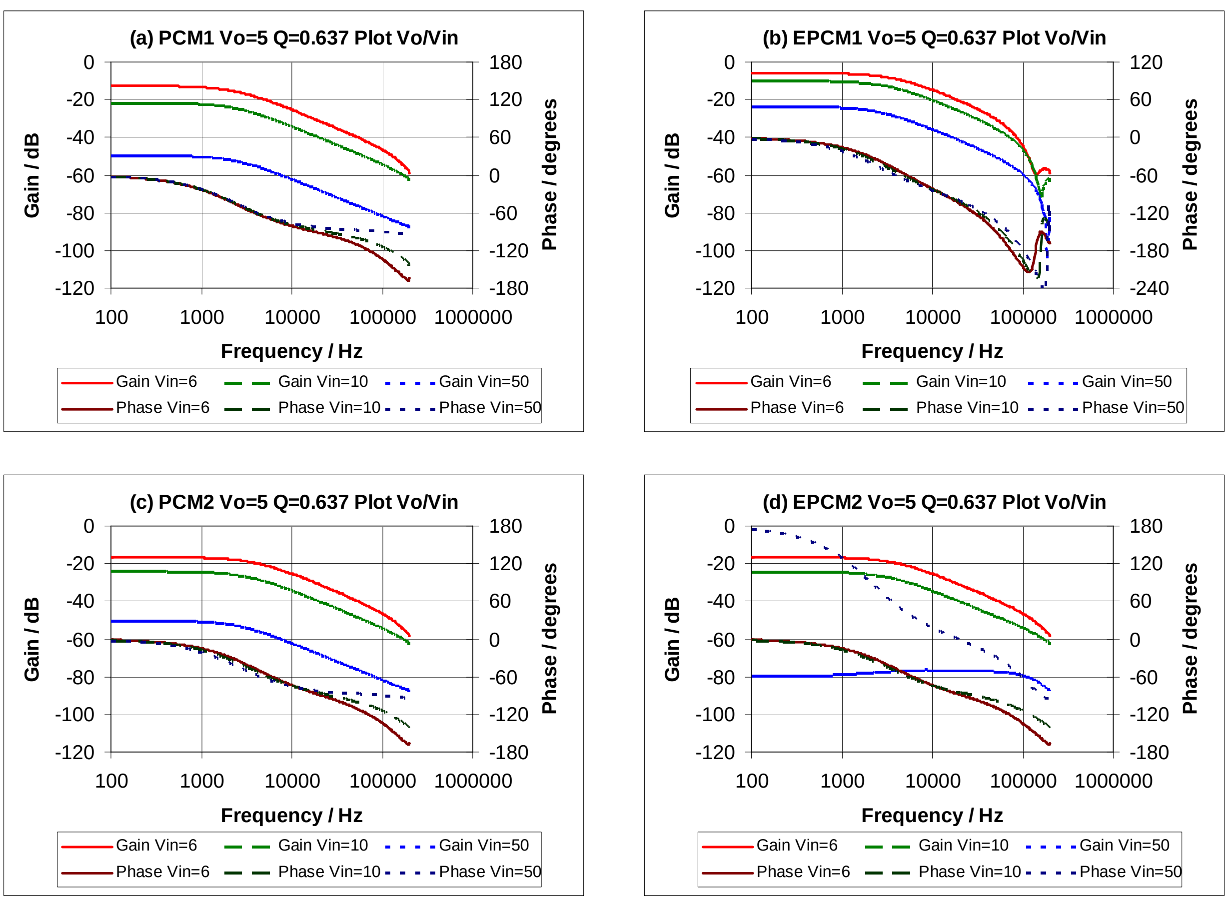


Figure 8: SIMPLIS SPICE results of line to output gain.

TABLE 3A

SUMMARY OF GAIN PARAMETERS FOR AVERAGED MODEL $S_f = \frac{V_O \cdot R_i}{L}$

Mode	S_e , S_n	F_m , K_{mp}	K_I , K_O
PCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot S_n + S_e) \cdot T}$ $K_{mp} = \frac{1}{0.5 \cdot R_i \cdot \frac{T}{L} \cdot D' + \frac{V_{SL}}{V_{IN}}}$	$K_I = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$ $K_O = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$
PCM2	$S_e = \frac{V_O \cdot K_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot S_n + S_e) \cdot T}$ $K_{mp} = \frac{1}{0.5 \cdot R_i \cdot \frac{T}{L} \cdot D' + K_{SL} \cdot D}$	$K_I = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$ $K_O = \left(0.5 \cdot R_i \cdot \frac{T}{L} - K_{SL}\right) \cdot D$
VCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot S_n + S_e) \cdot T}$ $K_{mp} = \frac{1}{0.5 \cdot R_i \cdot \frac{T}{L} \cdot D + \frac{V_{SL}}{V_{IN}}}$	$K_I = 0$ $K_O = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'$
VCM2	$S_e = \frac{(V_{IN} - V_O) \cdot K_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot S_n + S_e) \cdot T}$ $K_{mp} = \frac{1}{0.5 \cdot R_i \cdot \frac{T}{L} \cdot D + K_{SL} \cdot D'}$	$K_I = -K_{SL} \cdot D'$ $K_O = \left(0.5 \cdot R_i \cdot \frac{T}{L} - K_{SL}\right) \cdot D'$
EPCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot (S_f - S_n) + S_e) \cdot T}$ $K_{mp} = \frac{1}{\frac{V_{SL}}{V_{IN}} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'}$	$K_I = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$ $K_O = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$
EPCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot (S_f - S_n) + S_e) \cdot T}$ $K_{mp} = \frac{1}{K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'}$	$K_I = \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$ $K_O = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$
EVCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F_m = \frac{1}{(S_e - 0.5 \cdot S_f) \cdot T}$ $K_{mp} = \frac{1}{\frac{V_{SL}}{V_{IN}} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D}$	$K_I = 0$ $K_O = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'$
EVCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F_m = \frac{1}{(S_e - 0.5 \cdot S_f) \cdot T}$ $K_{mp} = \frac{1}{K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D}$	$K_I = -K_{SL} \cdot D'$ $K_O = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'$

TABLE 3B

SUMMARY OF GAIN PARAMETERS FOR AVERAGED MODEL $S_f = \frac{V_O \cdot R_i}{L}$

Mode	S_e, S_n	F_m, K_{mp}	K_I, K_O
VCM3	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot S_n + S_e) \cdot T}$ $K_{mp} = \frac{1}{0.5 \cdot D \cdot R_i \cdot \frac{T}{L} + K_{SL}}$	$K_I = -K_{SL} \cdot D'$ $K_O = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'$
EPCM3	$S_e = \frac{(V_{IN} - V_O) \cdot K_{SL} + V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot (S_f - S_n) + S_e) \cdot T}$ $K_{mp} = \frac{1}{(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}) \cdot D' + \frac{V_{SL}}{V_{IN}}}$	$K_I = \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$ $K_O = \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$
EPCM4	$S_e = \frac{V_{IN} \cdot K_{SL} + V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F_m = \frac{1}{(0.5 \cdot (S_f - S_n) + S_e) \cdot T}$ $K_{mp} = \frac{1}{K_{SL} + \frac{V_{SL}}{V_{IN}} - 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'}$	$K_I = \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$ $K_O = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$

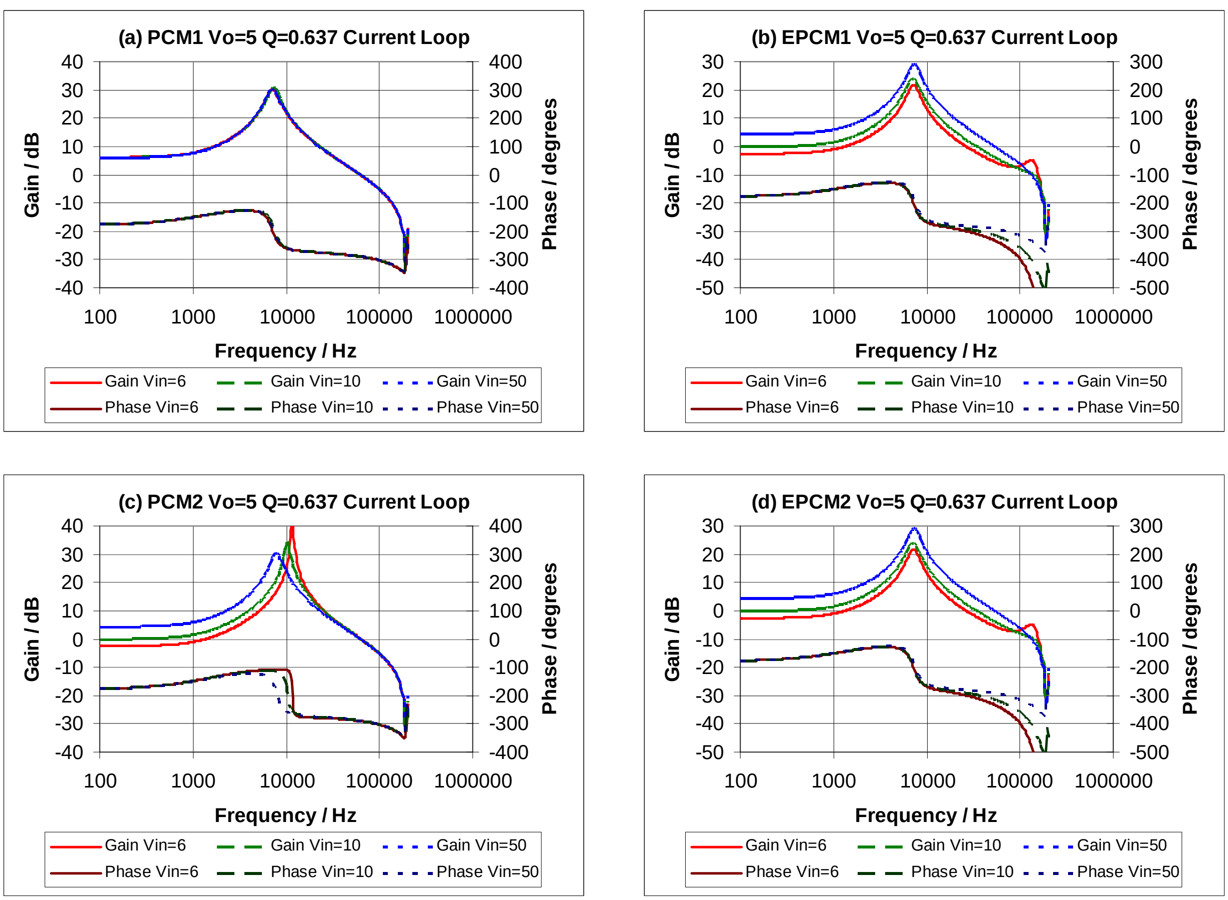


Figure 9: SIMPLIS SPICE results of current loop gain.

TABLE 4A
SUMMARY OF GAIN PARAMETERS FOR CONTINUOUS-TIME MODEL

Mode	S_e , S_n	F'_m , K'_{mp}	K'_f , K'_r
PCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$F'_m = \frac{1}{(S_n + S_e) \cdot T}$ $K'_{mp} = \frac{1}{R_i \cdot \frac{T}{L} \cdot D' + \frac{V_{SL}}{V_{IN}}}$	$K'_f = R_i \cdot \frac{T}{L} \cdot D \cdot (0.5 \cdot D - 1)$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2$
PCM2	$S_e = \frac{V_O \cdot K_{SL}}{T}$ $S_n = \frac{(V_{IN} - V_O) \cdot R_i}{L}$	$F'_m = \frac{1}{(S_n + S_e) \cdot T}$ $K'_{mp} = \frac{1}{R_i \cdot \frac{T}{L} \cdot D' + K_{SL} \cdot D}$	$K'_f = R_i \cdot \frac{T}{L} \cdot D \cdot (0.5 \cdot D - 1)$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2 - K_{SL} \cdot D$
VCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$F'_m = \frac{1}{(S_n + S_e) \cdot T}$ $K'_{mp} = \frac{1}{R_i \cdot \frac{T}{L} \cdot D + \frac{V_{SL}}{V_{IN}}}$	$K'_f = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (1 - D^2)$
VCM2	$S_e = \frac{(V_{IN} - V_O) \cdot K_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$F'_m = \frac{1}{(S_n + S_e) \cdot T}$ $K'_{mp} = \frac{1}{R_i \cdot \frac{T}{L} \cdot D + K_{SL} \cdot D'}$	$K'_f = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2 + K_{SL} \cdot D'$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (1 - D^2)$
EPCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F'_m = \frac{1}{S_e \cdot T}$ $K'_{mp} = \frac{1}{V_{SL} / V_{IN}}$	$K'_f = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2$
EPCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F'_m = \frac{1}{S_e \cdot T}$ $K'_{mp} = \frac{1}{K_{SL}}$	$K'_f = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2 - K_{SL} \cdot D$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2 - K_{SL} \cdot D$
EVCM1	$S_e = \frac{V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F'_m = \frac{1}{S_e \cdot T}$ $K'_{mp} = \frac{1}{V_{SL} / V_{IN}}$	$K'_f = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2$ $K'_r = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2$
EVCM2	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F'_m = \frac{1}{S_e \cdot T}$ $K'_{mp} = \frac{1}{K_{SL}}$	$K'_f = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2 + K_{SL} \cdot D'$ $K'_r = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2 + K_{SL} \cdot D'$

TABLE 4B
SUMMARY OF GAIN PARAMETERS FOR CONTINUOUS-TIME MODEL

Mode	S_e, S_n	F'_m, K'_{mp}	K'_f, K'_r
VCM3	$S_e = \frac{V_{IN} \cdot K_{SL}}{T}$ $S_n = \frac{V_O \cdot R_i}{L}$	$F'_m = \frac{1}{(S_n + S_e) \cdot T}$ $K'_{mp} = \frac{1}{R_i \cdot \frac{T}{L} \cdot D + K_{SL}}$	$K'_f = -0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2 + K_{SL} \cdot D'$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (1 - D^2) + K_{SL} \cdot D'$
EPCM3	$S_e = \frac{(V_{IN} - V_O) \cdot K_{SL} + V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F'_m = \frac{1}{S_e \cdot T}$ $K'_{mp} = \frac{1}{K_{SL} \cdot D' + \frac{V_{SL}}{V_{IN}}}$	$K'_f = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2 - K_{SL} \cdot D$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2$
EPCM4	$S_e = \frac{V_{IN} \cdot K_{SL} + V_{SL}}{T}$ $S_n = \frac{V_{IN} \cdot R_i}{L}$	$F'_m = \frac{1}{S_e \cdot T}$ $K'_{mp} = \frac{1}{K_{SL} + \frac{V_{SL}}{V_{IN}}}$	$K'_f = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot D^2 - K_{SL} \cdot D$ $K'_r = 0.5 \cdot R_i \cdot \frac{T}{L} \cdot (D')^2 - K_{SL} \cdot D$

TABLE 5
Calculated General Gain Parameters
V_{IN}=10 V_O=5 D=0.5 R_O=1 R_i=0.1 T=5µ L=5µ

MODE	V_{SLOPE}	K_{SL}	V_{SL}	K_m	K_n	m_c	Q
PCM1	V_{SL}	0.10	0.50	20.00	0.025	2.000	0.637
PCM2	$V_O \cdot K_{SL}$	0.10	---	10.00	0.025	2.000	0.637
VCM1	V_{SL}	0.10	0.50	20.00	0.075	2.000	0.637
VCM2	$(V_{IN}-V_O) \cdot K_{SL}$	0.10	---	10.00	0.175	2.000	0.637
EPCM1	V_{SL}	0.10	1.00	10.00	0.125	1.000	0.637
EPCM2	$V_{IN} \cdot K_{SL}$	0.10	---	10.00	0.025	1.000	0.637
EVCM1	V_{SL}	0.10	1.00	10.00	0.075	1.000	0.637
EVCM2	$V_{IN} \cdot K_{SL}$	0.10	---	10.00	0.175	1.000	0.637
VCM3	$V_{IN} \cdot K_{SL}$	0.10	---	10.00	0.225	3.000	0.318
EPCM3	$(V_{IN}-V_O) \cdot K_{SL} + V_{SL}$	0.10	0.50	20.00	0.025	1.000	0.637
EPCM4	$V_{IN} \cdot K_{SL} + V_{SL}$	0.05	0.50	10.00	0.075	1.000	0.637
PCM2	$0.5 \cdot V_O \cdot K_{SL}$	0.10	---	20.00	0.000	1.250	2.546
VCM2	$0.5 \cdot (V_{IN}-V_O) \cdot K_{SL}$	0.10	---	20.00	0.100	1.250	2.546

Table Notation:

PCM – Peak Current Mode
VCM – Valley Current Mode
EPCM – Emulated Peak Current Mode
EVCM – Emulated Valley Current Mode
1 – Fixed slope compensation V_{SL}
2 – Optimal slope compensation proportional to K_{SL}
3, 4 – Other fixed or proportional slope compensation implementations

In the last section of Table 5 for PCM2, peak current mode with a slope compensating ramp equal to one half the down slope of the inductor current represents a special case. K_n=0 which corresponds to infinite line rejection. This is difficult to achieve due to finite resistance and parasitic elements in a practical circuit. This is not optimal since sub-harmonic oscillation is possible at D>0.5 with feedback control. See section 4 on general slope compensation requirements.

TABLE 6
Calculated Values and Measured PSPICE Switching Circuit Data
V_{IN}=6 V_O=5 D=0.83 R_O=1 R_i=0.1 T=5µ L=5µ

				Calculated		Measured	
MODE	V_{SLOPE}	K_{SL}	V_{SL}	V_O/V_C	V_O/V_{IN}	V_O/V_C	V_O/V_{IN}
PCM1	V_{SL}	0.10	0.50	6.67	0.231	6.66	0.240
PCM2	$V_O \cdot K_{SL}$	0.10	---	4.29	0.149	4.25	0.154
VCM1	V_{SL}	0.10	0.10	6.67	0.324	6.55	0.325
VCM2	$(V_{IN}-V_O) \cdot K_{SL}$	0.10	---	6.00	0.392	5.98	0.383
EPCM1	V_{SL}	0.10	0.60	4.29	0.506	4.24	0.502
EPCM2	$V_{IN} \cdot K_{SL}$	0.10	---	4.29	0.149	4.25	0.148
EVCM1	V_{SL}	0.10	0.60	6.00	0.292	6.00	0.300
EVCM2	$V_{IN} \cdot K_{SL}$	0.10	---	6.00	0.392	5.96	0.388
VCM3	$V_{IN} \cdot K_{SL}$	0.10	---	4.29	0.577	4.23	0.571
EPCM3	$(V_{IN}-V_O) \cdot K_{SL} + V_{SL}$	0.10	0.50	6.67	0.231	6.61	0.231
EPCM4	$V_{IN} \cdot K_{SL} + V_{SL}$	0.05	0.50	3.75	0.391	3.71	0.389

TABLE 7
Calculated Values and Measured PSPICE Switching Circuit Data
V_{IN}=10 V_O=5 D=0.5 R_O=1 R_i=0.1 T=5µ L=5µ

				Calculated		Measured	
MODE	V_{SLOPE}	K_{SL}	V_{SL}	V_O/V_C	V_O/V_{IN}	V_O/V_C	V_O/V_{IN}
PCM1	V_{SL}	0.10	0.50	6.67	0.083	6.65	0.088
PCM2	$V_O \cdot K_{SL}$	0.10	---	5.00	0.063	5.00	0.065
VCM1	V_{SL}	0.10	0.50	6.67	0.250	6.57	0.249
VCM2	$(V_{IN}-V_O) \cdot K_{SL}$	0.10	---	5.00	0.438	4.97	0.427
EPCM1	V_{SL}	0.10	1.00	5.00	0.313	5.00	0.313
EPCM2	$V_{IN} \cdot K_{SL}$	0.10	---	5.00	0.063	5.00	0.063
EVCM1	V_{SL}	0.10	1.00	5.00	0.188	4.99	0.189
EVCM2	$V_{IN} \cdot K_{SL}$	0.10	---	5.00	0.438	4.99	0.430
VCM3	$V_{IN} \cdot K_{SL}$	0.10	---	5.00	0.563	4.98	0.550
EPCM3	$(V_{IN}-V_O) \cdot K_{SL} + V_{SL}$	0.10	0.50	6.67	0.083	6.56	0.085
EPCM4	$V_{IN} \cdot K_{SL} + V_{SL}$	0.05	0.50	5.00	0.188	5.00	0.188

TABLE 8
Calculated Values and Measured PSPICE Switching Circuit Data
V_{IN}=50 V_O=5 D=0.1 R_O=1 R_i=0.1 T=5µ L=5µ

				Calculated		Measured	
MODE	V_{SLOPE}	K_{SL}	V_{SL}	V_O/V_C	V_O/V_{IN}	V_O/V_C	V_O/V_{IN}
PCM1	V_{SL}	0.10	0.50	6.67	0.003	6.56	0.009
PCM2	$V_O \cdot K_{SL}$	0.10	---	6.25	0.003	6.17	0.009
VCM1	V_{SL}	0.10	4.50	6.67	0.063	6.52	0.063
VCM2	$(V_{IN}-V_O) \cdot K_{SL}$	0.10	---	4.17	0.415	4.16	0.407
EPCM1	V_{SL}	0.10	5.00	6.25	0.066	6.19	0.066
EPCM2	$V_{IN} \cdot K_{SL}$	0.10	---	6.25	0.003	6.14	0.005
EVCM1	V_{SL}	0.10	5.00	4.17	0.040	4.15	0.041
EVCM2	$V_{IN} \cdot K_{SL}$	0.10	---	4.17	0.415	4.16	0.412
VCM3	$V_{IN} \cdot K_{SL}$	0.10	---	6.25	0.628	6.21	0.618
EPCM3	$(V_{IN}-V_O) \cdot K_{SL} + V_{SL}$	0.10	0.50	6.67	0.003	6.53	0.005
EPCM4	$V_{IN} \cdot K_{SL} + V_{SL}$	0.05	0.50	8.33	0.013	8.09	0.014

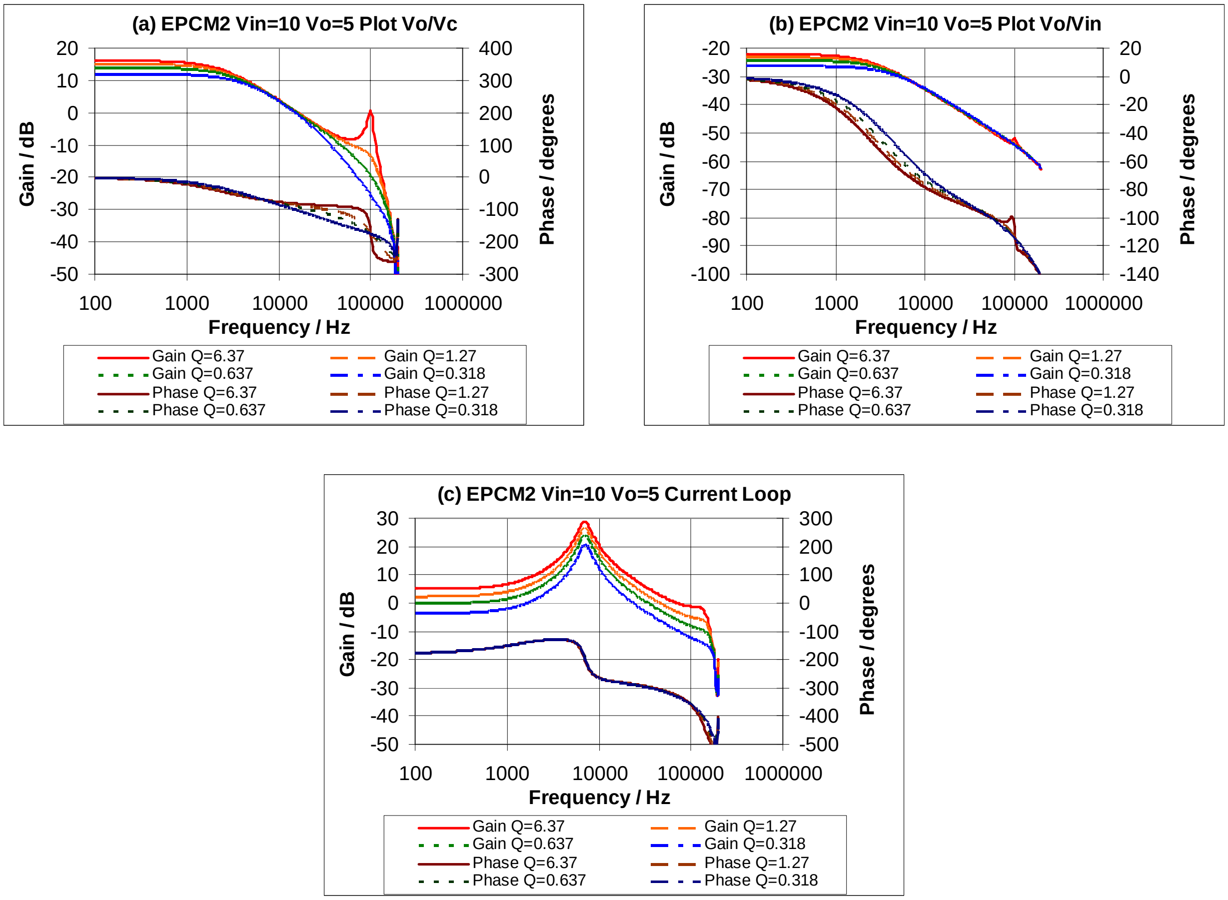
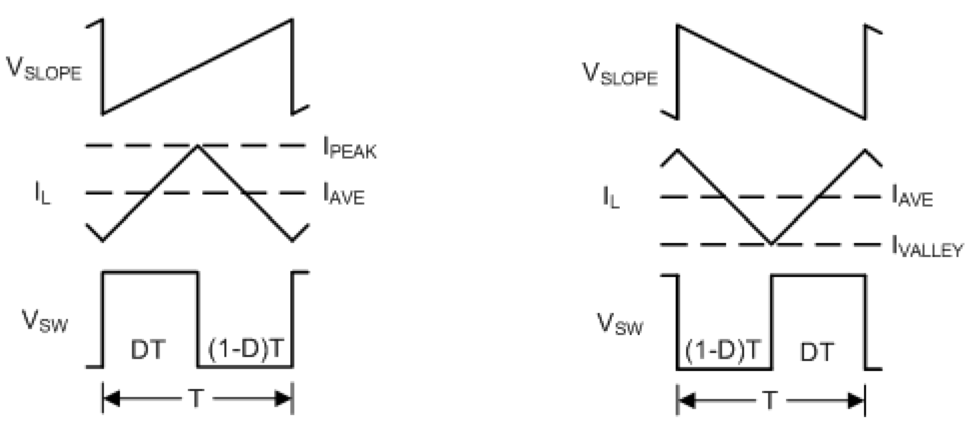


Figure 10: SIMPLIS SPICE results for EPCM2 with varying Q.

3. Discrete Time Analysis for Slope Compensation



(a) Peak current mode

(b) Valley current mode

Figure 11: Slope compensation for peak and valley current mode buck.

Peak Current Mode
Trailing Edge Modulation

For peak current mode control, the peak current must be accounted for in its relationship to the average current. By perturbing the duty cycle, we can see the effect on V_C.

Write the equation for the voltage at V_C:

$$i_L \cdot R_i + 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + V_{SLOPE} \cdot d = v_C \tag{44}$$

V_{IN}, V_O, and V_{SLOPE} are considered to be fixed with respect to the period T. The duty cycle is perturbed, which shows the effect on the control voltage with respect to the inductor current. The quantity $\Delta I_L = \hat{i}_L(T)$, $\Delta D = \hat{d}(T)$ and $\Delta V_C = \hat{v}_C(T)$.

$$\Delta I_L \cdot R_i + 0.5 \cdot (V_{IN} - V_O) \cdot \Delta D \cdot \frac{T}{L} \cdot R_i + V_{SLOPE} \cdot \Delta D = \Delta V_C \tag{45}$$

To maintain constant average inductor current, the control voltage must change by ½ the ripple current times the current sense gain:

$$\Delta V_C = -0.5 \cdot \Delta I_{PK} \cdot R_i \tag{46}$$

$$\Delta I_{PK} = V_O \cdot (1 - \Delta D) \cdot \frac{T}{L} = -V_O \cdot \Delta D \cdot \frac{T}{L} \tag{47}$$

Combine equations and solve for $\Delta D / \Delta I_L$:

$$\frac{\Delta D}{\Delta I_L} = -\frac{R_i}{V_{IN}} \cdot \frac{1}{(0.5 - D) \cdot R_i \cdot \frac{T}{L} + V_{SLOPE} / V_{IN}} \tag{48}$$

The term $\dfrac{1}{(0.5 - D) \cdot R_i \cdot \frac{T}{L} + V_{SLOPE} / V_{IN}}$ is equivalent to the peak current mode modulator gain K_m with fixed V_{SLOPE}.

By factoring, this becomes:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot \left[\left(1 + \frac{V_{SLOPE} / T}{(V_{IN} - V_O) \cdot R_i / L} \right) \cdot (1 - D) - 0.5 \right]} \tag{49}$$

This can be expressed as:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot (m_c \cdot D' - 0.5)} \tag{50}$$

Where: $D' = 1 - D$ $\quad m_c = 1 + \dfrac{S_e}{S_n}$

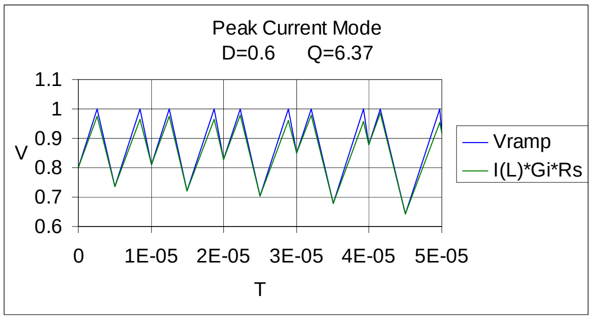
Slope compensating ramp: $S_e = V_{SLOPE} / T$
Positive current sense ramp: $S_n = (V_{IN} - V_O) \cdot R_i / L$

Reviewing the equation for $\Delta D/\Delta I_L$, when D>0.5, the T/L term goes negative. V_{SLOPE} must be large enough to compensate, or this will manifest itself in the time domain as a sub-harmonic oscillation of the current loop. If the condition is set such that:

$$D \cdot \frac{T}{L} = \frac{V_{SLOPE}}{V_{IN} \cdot R_i} \tag{51}$$

Then $m_C \cdot D' = 1$, so the current loop will be stable for any duty cycle. To meet this condition, solve for V_{SLOPE}:

$$V_{SLOPE} = V_O \cdot R_i \cdot \frac{T}{L} \tag{52}$$



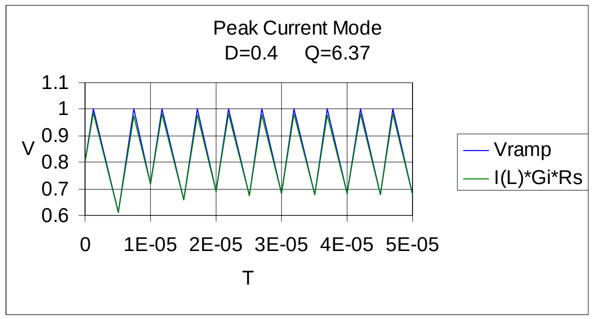


Figure 12: Peak current mode sub-harmonic oscillation. For D<0.5, sub-harmonic oscillation is damped. For D>0.5, sub-harmonic oscillation builds with insufficient slope compensation.

Valley Current Mode
Leading Edge Modulation

For valley current mode control, the valley current must be accounted for in its relationship to the average current. By perturbing the duty cycle, we can see the effect on V_C.

Write the equation for the voltage at V_C:

$$i_L \cdot R_i - 0.5 \cdot v_O \cdot (1-d) \cdot \frac{T}{L} \cdot R_i - V_{SLOPE} \cdot (1-d) = v_C \tag{53}$$

V_{IN}, V_O, and V_{SLOPE} are considered to be fixed with respect to the period T. The duty cycle is perturbed, which shows the effect on the control voltage with respect to the inductor current. The quantity $\Delta I_L = \hat{i}_L(T)$, $\Delta D = \hat{d}(T)$ and $\Delta V_C = \hat{v}_C(T)$.

$$\Delta I_L \cdot R_i + 0.5 \cdot V_O \cdot \Delta D \cdot \frac{T}{L} \cdot R_i + V_{SLOPE} \cdot \Delta D = \Delta V_C \tag{54}$$

To maintain constant average inductor current, the control voltage must change by ½ the ripple current:

$$\Delta V_C = 0.5 \cdot \Delta I_{PK} \cdot R_i \tag{55}$$

$$\Delta I_{PK} = (V_{IN} - V_O) \cdot \Delta D \cdot \frac{T}{L} \tag{56}$$

Combine equations and solve for $\Delta D/\Delta I_L$:

$$\frac{\Delta D}{\Delta I_L} = -\frac{R_i}{V_{IN}} \cdot \frac{1}{(D-0.5) \cdot R_i \cdot \frac{T}{L} + V_{SLOPE} / V_{IN}} \qquad (57)$$

The term $\frac{1}{(D-0.5) \cdot R_i \cdot \frac{T}{L} + V_{SLOPE} / V_{IN}}$ is equivalent to the valley current mode modulator gain K_m with fixed V_{SLOPE}.

By factoring, this becomes:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot \left[\left(1 + \frac{V_{SLOPE} / T}{V_O \cdot R_i / L}\right) \cdot D - 0.5\right]} \qquad (58)$$

This can be expressed as:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot (m_c \cdot D - 0.5)} \qquad (59)$$

Where: $m_c = 1 + \frac{S_e}{S_n}$

Slope compensating ramp: $S_e = V_{SLOPE} / T$

Negative current sense ramp: $S_n = V_O \cdot R_i / L$

Reviewing the equation for $\Delta D/\Delta I_L$, when D<0.5, the T/L term goes negative. V_{SLOPE} must be large enough to compensate, or this will manifest itself in the time domain as a sub-harmonic oscillation of the current loop. If the condition is set such that:

$$(1-D) \cdot \frac{T}{L} = \frac{V_{SLOPE}}{V_{IN} \cdot R_i} \qquad (60)$$

Then $m_C \cdot D = 1$, so the current loop will be stable for any duty cycle. To meet this condition, solve for V_{SLOPE}.

$$V_{SLOPE} = (V_{IN} - V_O) \cdot R_i \cdot \frac{T}{L} \qquad (61)$$

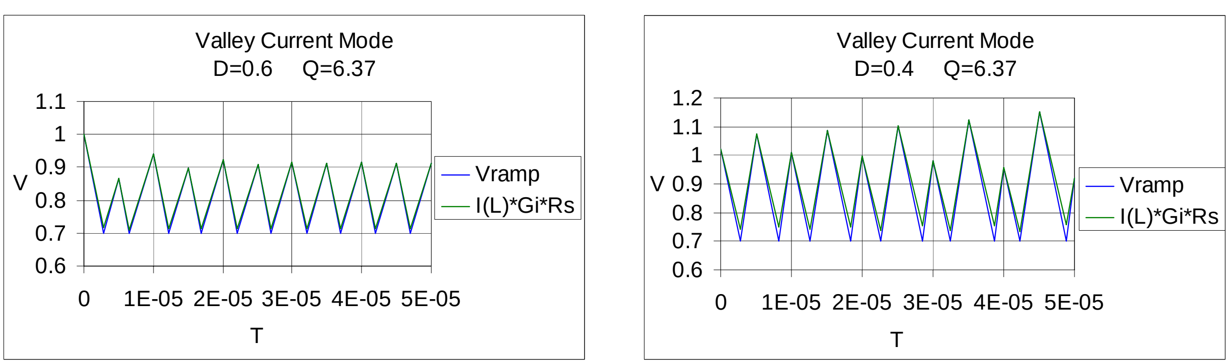


Figure 13: Valley current mode sub-harmonic oscillation. For D>0.5, sub-harmonic oscillation is damped. For D<0.5, sub-harmonic oscillation builds with insufficient slope compensation.

Emulated Peak Current Mode
Valley Sample and Hold

The valley current is sampled on the down slope of the inductor current. This is used as the DC value of current to start the next cycle. A slope compensating ramp is added to produce V_{RAMP} at the modulator input.

Write the equation for the voltage at V_C:

$$i_L \cdot R_i - 0.5 \cdot (v_{IN} - v_O) \cdot d \cdot \frac{T}{L} \cdot R_i + V_{SLOPE} \cdot d = v_C \qquad (62)$$

V_{IN}, V_O, and V_{SLOPE} are considered to be fixed with respect to the period T. The duty cycle is perturbed, which shows the effect on the control voltage with respect to the inductor current. The quantity $\Delta I_L = \hat{i}_L(T)$, $\Delta D = \hat{d}(T)$ and $\Delta V_C = \hat{v}_C(T)$.

$$\Delta I_L \cdot R_i - 0.5 \cdot (V_{IN} - V_O) \cdot \Delta D \cdot \frac{T}{L} \cdot R_i + V_{SLOPE} \cdot \Delta D = \Delta V_C \qquad (63)$$

To maintain constant average inductor current, the control voltage must change by ½ the ripple current:

$$\Delta V_C = -0.5 \cdot \Delta I_{PK} \cdot R_i \qquad (64)$$

$$\Delta I_{PK} = V_O \cdot (1 - \Delta D) \cdot \frac{T}{L} = -V_O \cdot \Delta D \cdot \frac{T}{L} \qquad (65)$$

Combine equations and solve for $\Delta D/\Delta I_L$:

$$\frac{\Delta D}{\Delta I_L} = -\frac{R_i}{V_{IN}} \cdot \frac{1}{V_{SLOPE}/V_{IN} - 0.5 \cdot R_i \cdot \frac{T}{L}} \qquad (66)$$

By factoring, this becomes:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot \left(\frac{V_{SLOPE}/T}{V_{IN} \cdot R_i / L} - 0.5 \right)} \qquad (67)$$

This can also be written as:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot (m_C - 0.5)} \qquad (68)$$

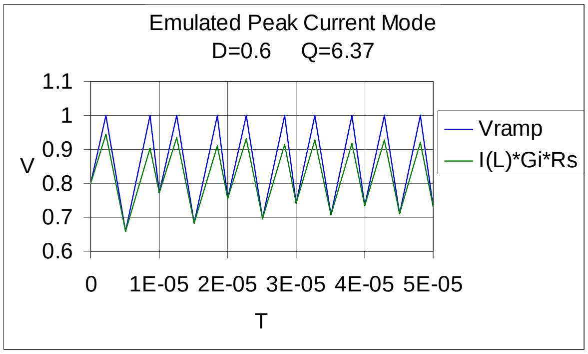
Where: $m_C = \frac{S_e}{S_n}$ $\qquad S_e = V_{SLOPE} / T$ $\qquad S_n = V_{IN} \cdot R_i / L$

Reviewing the equation for $\Delta D/\Delta I_L$, when $m_C<0.5$, the T/L term goes negative. V_{SLOPE} must be large enough to compensate, or this will manifest itself in the time domain as a sub-harmonic oscillation of the current loop. If the condition is set such that:

$$\frac{V_{SLOPE}}{T} = \frac{V_{IN} \cdot R_i}{L} \qquad (69)$$

Then $m_C = 1$, so the current loop will be unconditionally stable. Unlike peak or valley current mode, slope compensation is independent of duty cycle. To meet this condition, solve for V_{SLOPE}.

$$V_{SLOPE} = V_{IN} \cdot R_i \cdot \frac{T}{L} \tag{70}$$



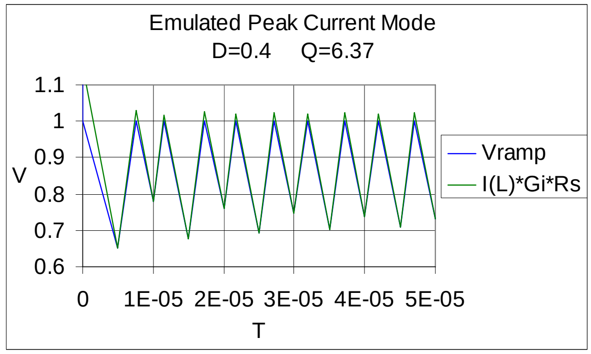


Figure 14: Emulated peak current mode sub-harmonic oscillation. Tendency for sub-harmonic oscillation is independent of duty cycle. Even with minimal damping, it will eventually die out.

Emulated Valley Current Mode
Peak Sample and Hold

The peak current is sampled on the positive slope of the inductor current. This is used as the DC value of current to start the next cycle. A slope compensating ramp is added to produce V_{RAMP} at the modulator input.

Write the equation for the voltage at V_C:

$$i_L \cdot R_i + 0.5 \cdot v_O \cdot (1-d) \cdot \frac{T}{L} \cdot R_i - V_{SLOPE} \cdot (1-d) = v_C \tag{71}$$

V_{IN}, V_O, and V_{SLOPE} are considered to be fixed with respect to the period T. The duty cycle is perturbed, which shows the effect on the control voltage with respect to the inductor current. The quantity $\Delta I_L = \hat{i}_L(T)$, $\Delta D = \hat{d}(T)$ and $\Delta V_C = \hat{v}_C(T)$.

$$\Delta I_L \cdot R_i - 0.5 \cdot V_O \cdot \Delta D \cdot \frac{T}{L} \cdot R_i + V_{SLOPE} \cdot \Delta D = \Delta V_C \tag{72}$$

To maintain constant average inductor current, the control voltage must change by ½ the ripple current:

$$\Delta V_C = 0.5 \cdot \Delta I_{PK} \cdot R_i \tag{73}$$

$$\Delta I_{PK} = (V_{IN} - V_O) \cdot \Delta D \cdot \frac{T}{L} \tag{74}$$

Combine equations and solve for $\Delta D / \Delta I_L$:

$$\frac{\Delta D}{\Delta I_L} = -\frac{R_i}{V_{IN}} \cdot \frac{1}{V_{SLOPE}/V_{IN} - 0.5 \cdot R_i \cdot \frac{T}{L}} \tag{75}$$

By factoring, this becomes:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot \left(\frac{V_{SLOPE}/T}{V_{IN} \cdot R_i / L} - 0.5 \right)} \tag{76}$$

This can be expressed as:

$$\frac{\Delta D}{\Delta I_L} = -\frac{1}{V_{IN}} \cdot \frac{1}{\frac{T}{L} \cdot (m_c - 0.5)} \tag{77}$$

Where: $m_C = \frac{S_e}{S_n}$ $\quad S_e = V_{SLOPE} / T$ $\quad S_n = V_{IN} \cdot R_i / L$

Reviewing the equation for $\Delta D/\Delta I_L$, when m_C<0.5, the T/L term goes negative. V_{SLOPE} must be large enough to compensate, or this will manifest itself in the time domain as a sub-harmonic oscillation of the current loop. If the condition is set such that:

$$\frac{V_{SLOPE}}{T} = \frac{V_{IN} \cdot R_i}{L} \tag{78}$$

Then m_C = 1, so the current loop will be unconditionally stable. Unlike peak or valley current mode, slope compensation is independent of duty cycle. To meet this condition, solve for V_{SLOPE}.

$$V_{SLOPE} = V_{IN} \cdot R_i \cdot \frac{T}{L} \tag{79}$$

4. General Slope Compensation Requirements

The graphs in the preceding section were made by entering the discrete time expressions for the inductor current and slope compensating ramp into an Excel spreadsheet. The results are plotted on a cycle by cycle basis. This shows the current loop behavior without any feedback to the control voltage. Reference [6] discusses the voltage loop gain effect on the slope compensation requirement for peak current mode. Peaking of the closed loop gain due to insufficient slope compensation and ripple on the control voltage can cause sub-harmonic oscillation before the calculated limit, i.e. at duty cycles below 0.5 for peak current mode.

For any mode of operation, when the sum of the sensed inductor current's slope (times the current sense gain) plus the slope of the compensation ramp is proportional to V_{IN}, any tendency toward sub-harmonic oscillation will damp in one switching cycle. This condition is represented by equations 52, 61, 70 and 79, which corresponds to a Q of 0.637. Operation is considered to be optimal, in that the effective sampled gain inductor pole is fixed in frequency with respect to changes in line voltage. This allows for the highest closed loop gain without any tendency toward sub-harmonic oscillation. Increasing the external ramp beyond this point will lower the modulator gain, consequently shifting toward a more voltage mode behavior.

The effective sampled gain inductor pole frequency (45° phase shift) is given by:

$$f_L(Q) = \frac{1}{4 \cdot T \cdot Q} \cdot \left(\sqrt{1 + 4 \cdot Q^2} - 1 \right) \tag{80}$$

5. Correlation of Measured Data

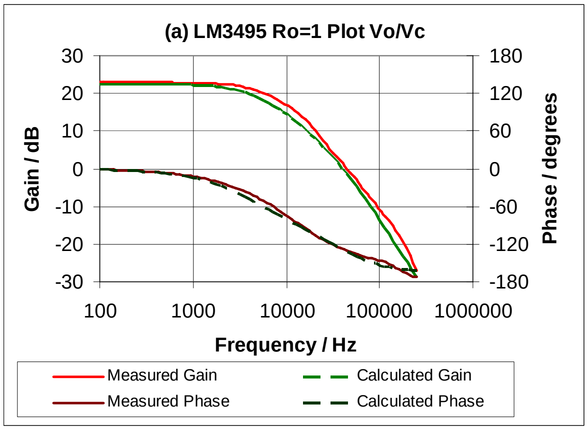
The LM3495 standard reference design was chosen as a platform for correlation. It is an emulated peak current mode controller with MOSFET current sensing and internally generated slope compensation using mode EPCM4. Operating parameters:

V_{IN}=12	V_O=1.2	T=2μs	V_{SL}=0.125	K_{SL}=1/16	L=1μH
R_L=3mΩ	G_I=4	R_S=3.4mΩ	C_O=200μF	R_C=1mΩ	R_O=1Ω and 0.2Ω

Measurement of control to output gain was made using an AP200 frequency response analyzer.

The simplified transfer function using equation 41 was entered into SPICE as LaPlace for the calculated results. Nominal data sheet values were used for the components and operating parameters.

Comparison of the results is shown in figure 15.



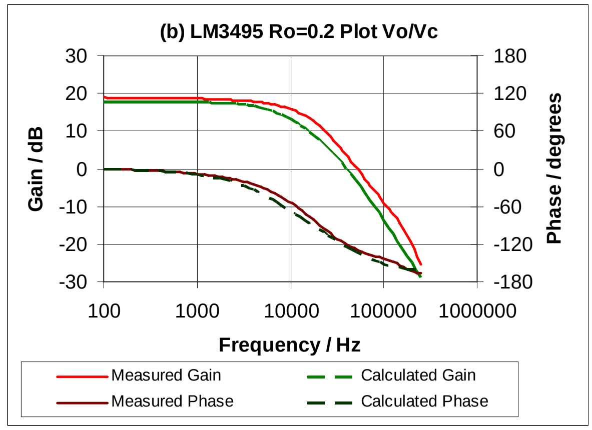


Figure 15: LM3495 control to output gain.

6. Conclusion

This analysis has focused on current mode buck regulators with continuous inductor current. Using the methodology outlined here, general expressions for the boost and buck-boost can be developed. Preliminary work has demonstrated good correlation between switching and linear models. Using the gain coefficients from reference [2], general expressions for discontinuous conduction mode can also be developed.

Limitations of existing models have been identified, with direction for further work. Regardless of the limitations, the simplified transfer functions and general gain parameters allow accurate modeling of the control to output gain. This is the most important parameter from a design standpoint. Tools like SIMPLIS provide bode plots for any transfer function directly from the switching model.

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[2] R.B. Ridley, "*A New Continuous-Time Model for Current-Mode Control with Constant Frequency, Constant On-Time, and Constant Off-Time, in CCM and DCM,*" IEEE PESC Record, pp. 382–389, 1990.

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[6] Barney Holland, "*Modeling, Analysis and Compensation of the Current-Mode Converter,*" Powercon 11 Proceedings, I-2, 1984.

Notes:

EMULATED CURRENT MODE CONTROL FOR BUCK REGULATORS USING SAMPLE AND HOLD TECHNIQUE

Small Signal Linear Analysis and Comparison to Peak and Valley Methods

by

Robert Sheehan
Principal Applications Engineer
National Semiconductor Corporation
Santa Clara, CA

APPENDIX A

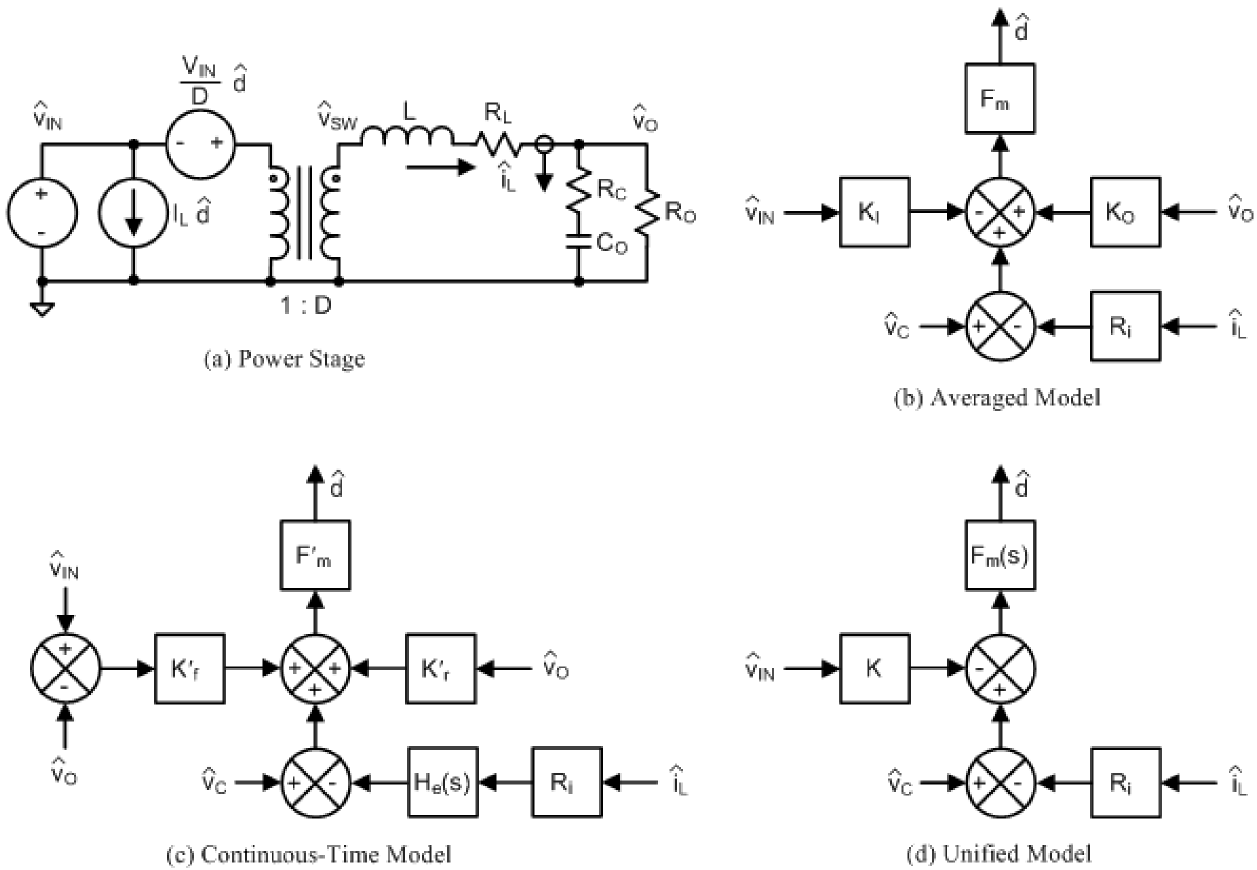

Figure A-1: Buck regulator linear models.

Define Z_O and Z_L:

$$Z_O = \left(\frac{1}{s \cdot C_O} + R_C\right) \| R_O = \frac{R_O \cdot (1 + s \cdot C_O \cdot R_C)}{1 + s \cdot C_O \cdot (R_O + R_C)} \quad \text{(A.1)}$$

$$Z_L = s \cdot L + R_L + R_S \quad \text{(A.2)}$$

Averaged Model
Derivation of Transfer Functions Using General Gain Parameters

Starting with $\hat{v}_O$, write the transfer function in terms of voltages:

$$\hat{v}_O = \hat{v}_{SW} \cdot \frac{Z_O}{Z_O + Z_L} \tag{A.3}$$

$$\hat{v}_{SW} = \hat{v}_{IN} \cdot D + V_{IN} \cdot \hat{d} \tag{A.4}$$

$$\hat{d} = F_m \cdot (\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K_I + \hat{v}_O \cdot K_O) \tag{A.5}$$

$$\hat{i}_L = \frac{\hat{v}_O}{Z_O} \tag{A.6}$$

Combining equations A.3 through A.6 yields:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + V_{IN} \cdot F_m \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{Z_O} - \hat{v}_{IN} \cdot K_I + \hat{v}_O \cdot K_O\right)\right] \cdot \frac{Z_O}{Z_O + Z_L} \tag{A.7}$$

Define K_{mp} as:

$$K_{mp} = V_{IN} \cdot F_m \tag{A.8}$$

Setting $\hat{v}_{IN}$ to zero allows the control to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{Z_O}{Z_O \cdot \left(\frac{1}{K_{mp}} - K_O\right) + Z_L \cdot \frac{1}{K_{mp}} + R_i} \tag{A.9}$$

Define K_m as the modulator gain where:

$$K_m = \frac{1}{\frac{1}{K_{mp}} - K_O} \tag{A.10}$$

This allows the control to output gain to be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{Z_O}{\frac{Z_O}{K_m} + \frac{Z_L}{K_{mp}} + R_i} \tag{A.11}$$

Setting $\hat{v}_C$ to zero allows the line to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{D \cdot \left(\frac{1}{K_{mp}} - \frac{K_I}{D}\right) \cdot Z_O}{Z_O \cdot \left(\frac{1}{K_{mp}} - K_O\right) + Z_L \cdot \frac{1}{K_{mp}} + R_i} \tag{A.12}$$

Define K_n as the audio susceptibility coefficient:

$$K_n = \frac{1}{K_{mp}} - \frac{K_I}{D} \tag{A.13}$$

This allows the line to output gain to be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{D \cdot K_n \cdot Z_O}{\frac{Z_O}{K_m} + \frac{Z_L}{K_{mp}} + R_i} \tag{A.14}$$

The denominator of the closed loop gain expressions is also used for the control to inductor current gain. Formal derivation is done by setting $\hat{v}_{IN} = 0$ and $\hat{v}_O = \hat{i}_L \cdot Z_O$.

$$\frac{\hat{i}_L}{\hat{v}_C} = \frac{1}{\frac{Z_O}{K_m} + \frac{Z_L}{K_{mp}} + R_i} \tag{A.15}$$

Continuous-Time Model
Derivation of Transfer Functions Using General Gain Parameters

Starting with $\hat{d}$, write the transfer function in terms of voltages:

$$\hat{d} = F'_m \cdot (\hat{v}_C - \hat{i}_L \cdot R_i \cdot H_e(s) + \hat{v}_{ON} \cdot K'_f + \hat{v}_{OFF} \cdot K'_r) \tag{A.16}$$

Combining with equations A.3, A.4 and A.6 for the power stage yields:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + V_{IN} \cdot F'_m \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i \cdot H_e(s)}{Z_O} + \hat{v}_{ON} \cdot K'_f + \hat{v}_{OFF} \cdot K'_r\right)\right] \cdot \frac{Z_O}{Z_O + Z_L} \tag{A.17}$$

Define K'_{mp} as:

$$K'_{mp} = V_{IN} \cdot F'_m \tag{A.18}$$

Let $\hat{v}_{ON} = \hat{v}_{IN} - \hat{v}_O$, $\hat{v}_{OFF} = \hat{v}_O$. The transfer function becomes:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + K'_{mp} \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i \cdot H_e(s)}{Z_O} + \hat{v}_{IN} \cdot K'_f - \hat{v}_O \cdot K'_f + \hat{v}_O \cdot K'_r\right)\right] \cdot \frac{Z_O}{Z_O + Z_L} \tag{A.19}$$

Setting $\hat{v}_{IN}$ to zero allows the control to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{Z_O}{Z_O \cdot \left(\frac{1}{K'_{mp}} + K'_f - K'_r\right) + Z_L \cdot \frac{1}{K'_{mp}} + R_i \cdot H_e(s)} \tag{A.20}$$

Define K_m as the modulator gain where:

$$K_m = \frac{1}{\frac{1}{K'_{mp}} + K'_f - K'_r} \tag{A.21}$$

This allows the control to output gain to be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{Z_O}{\frac{Z_O}{K_m} + \frac{Z_L}{K'_{mp}} + R_i \cdot H_e(s)} \tag{A.22}$$

Setting $\hat{v}_C$ to zero allows the line to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{Z_O \cdot D \cdot \left(\frac{1}{K'_{mp}} + \frac{K'_f}{D} \right)}{Z_O \cdot \left(\frac{1}{K'_{mp}} + K'_f - K'_r \right) + Z_L \cdot \frac{1}{K'_{mp}} + R_i \cdot H_e(s)} \tag{A.23}$$

Define K_n as the audio susceptibility coefficient:

$$K_n = \frac{1}{K'_{mp}} + \frac{K'_f}{D} \tag{A.24}$$

This allows the line to output gain to be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{Z_O \cdot D \cdot K_n}{\frac{Z_O}{K_m} + \frac{Z_L}{K'_{mp}} + R_i \cdot H_e(s)} \tag{A.25}$$

The denominator of the closed loop gain expressions is also used for the control to inductor current gain. Formal derivation is done by setting $\hat{v}_{IN} = 0$ and $\hat{v}_O = \hat{i}_L \cdot Z_O$.

$$\frac{\hat{i}_L}{\hat{v}_C} = \frac{1}{\frac{Z_O}{K_m} + \frac{Z_L}{K'_{mp}} + R_i \cdot H_e(s)} \tag{A.26}$$

Using the closed loop denominator term, define a single sampling gain block which incorporates K'_{mp} inside the current loop. Let:

$$\frac{Z_O}{K_m} + \frac{Z_L}{K_m} + R_i \cdot H(s) = \frac{Z_O}{K_m} + \frac{Z_L}{K'_{mp}} + R_i \cdot H_e(s) \tag{A.27}$$

The sampling gain term becomes:

$$H(s) = H_e(s) + \frac{Z_L}{R_i} \cdot \left(\frac{1}{K'_{mp}} - \frac{1}{K_m} \right) \tag{A.28}$$

Where: $H_e(s) = 1 + \frac{s}{\omega_n \cdot Q_z} + \frac{s^2}{\omega_n^{\,2}}$ $\omega_n = \frac{\pi}{T}$ $Q_z = -\frac{2}{\pi}$ (A.29)

The transfer functions can now be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{K_m \cdot Z_O}{Z_O + Z_L + K_m \cdot R_i \cdot H(s)} \quad \text{(A.30)}$$

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{D \cdot K_n \cdot K_m \cdot Z_O}{Z_O + Z_L + K_m \cdot R_i \cdot H(s)} \quad \text{(A.31)}$$

$$\frac{\hat{i}_L}{\hat{v}_C} = \frac{K_m}{Z_O + Z_L + K_m \cdot R_i \cdot H(s)} \quad \text{(A.32)}$$

The general transfer function is:

$$\hat{v}_O = K_m \cdot \left[\hat{v}_C - \hat{i}_L \cdot R_i \cdot H(s) + \hat{v}_{IN} \cdot D \cdot K_n\right] \cdot \frac{Z_O}{Z_O + Z_L} \quad \text{(A.33)}$$

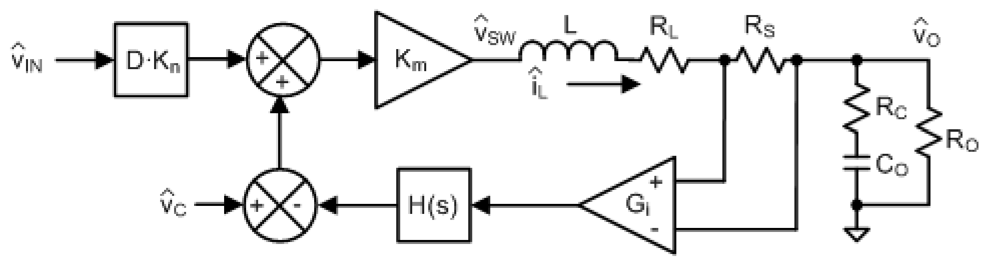

Figure A-2: Continuous-time model using general gain parameters. This model is valid for control to output transfer functions of all operating modes. The line to output transfer function is valid at dc, but diverges from the actual response over frequency.

Unified Model
Derivation of Transfer Functions Using General Gain Parameters

Starting with $\hat{d}$, write the transfer function in terms of voltages:

$$\hat{d} = F_m(s) \cdot (\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K) \quad \text{(A.34)}$$

Combining with equations A.3, A.4 and A.6 for the power stage yields:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + V_{IN} \cdot F_m(s) \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{Z_O} - \hat{v}_{IN} \cdot K\right)\right] \cdot \frac{Z_O}{Z_O + Z_L} \quad \text{(A.35)}$$

Define: $V_{IN} \cdot F_m(s) = K_m \cdot H_P(s)$ (A.36)

The transfer function becomes:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + K_m \cdot H_p(s) \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{Z_O} - \hat{v}_{IN} \cdot K\right)\right] \cdot \frac{Z_O}{Z_O + Z_L} \quad \text{(A.37)}$$

Setting $\hat{v}_{IN}$ to zero allows the control to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{K_m \cdot Z_O \cdot H_P(s)}{Z_O + Z_L + K_m \cdot R_i \cdot H_P(s)} \tag{A.38}$$

Setting $\hat{v}_C$ to zero allows the line to output gain to be found:

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{[D - K \cdot K_m \cdot H_P(s)] \cdot Z_O}{Z_O + Z_L + K_m \cdot R_i \cdot H_P(s)} \tag{A.39}$$

Setting $\hat{v}_{IN} = 0$ and $\hat{v}_O = \hat{i}_L \cdot Z_O$ allows the control to inductor current gain to be found:

$$\frac{\hat{i}_L}{\hat{v}_C} = \frac{K_m \cdot H_P(s)}{Z_O + Z_L + K_m \cdot R_i \cdot H_P(s)} \tag{A.40}$$

Setting $\hat{v}_C = 0$, $\hat{v}_{IN} = 0$ and $\hat{v}_O = \hat{i}_L \cdot Z_O$ allows the current loop gain to be found:

$$\frac{\hat{i}_L}{\hat{i}'_L} = -\frac{K_m \cdot R_i \cdot H_P(s)}{Z_O + Z_L} \tag{A.41}$$

Setting $H_P(s)=1$ and comparing the line to output gain to the averaged and continuous-time models:

$$D \cdot K_n \cdot K_m = D - K \cdot K_m \qquad \text{Therefore:} \qquad K = D \cdot \left(\frac{1}{K_m} - K_n\right) \tag{A.42}$$

The sampling gain term is defined as:

$$H_p(s) = \frac{1}{1 + \frac{s}{\omega_{Hp}}} \qquad \text{Where:} \quad \omega_{Hp} = \frac{\omega_n}{Q} \tag{A.43}$$

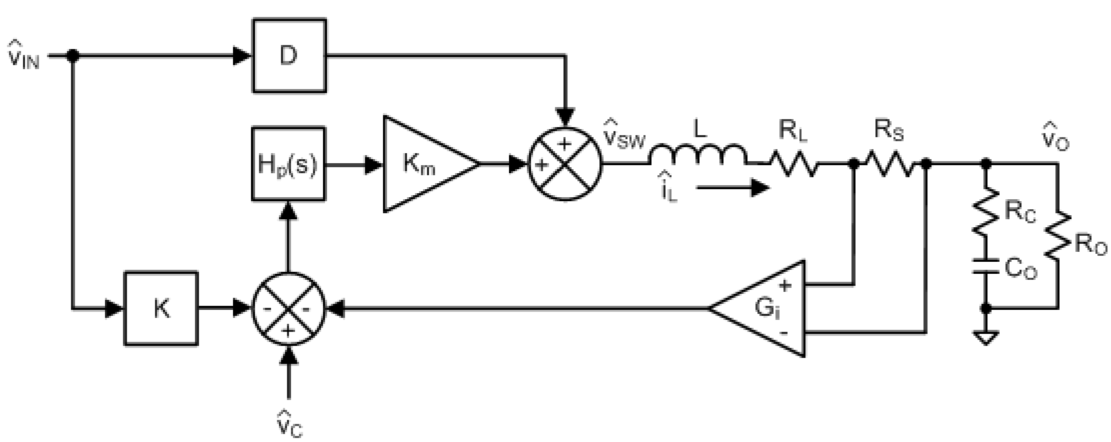

Figure A-3: Unified model uses the high frequency asymptote for a single pole in series with the modulator. This accurately models the current loop and control to output transfer functions, but is limited to PCM1, VCM1 and VCM3 operating modes.

Averaged Model
Derivation of Factored Pole/Zero Form

Starting with the control to output gain of equation A.11, expand Z_O and Z_L:

$$\frac{\hat{v}_O}{\hat{v}_C}=\frac{Z_O}{\frac{Z_O}{K_m}+\frac{Z_L}{K_{mp}}+R_i}=\frac{\frac{R_O\cdot(1+s\cdot C_O\cdot R_C)}{1+s\cdot C_O\cdot(R_O+R_C)}}{\frac{R_O\cdot(1+s\cdot C_O\cdot R_C)}{K_m\cdot(1+s\cdot C_O\cdot(R_O+R_C))}+\frac{s\cdot L+R_L+R_S}{K_{mp}}+R_i} \tag{A.44}$$

Rationalize the numerator and factor R_O/R_i:

$$\frac{\hat{v}_O}{\hat{v}_C}=\frac{R_O}{R_i}\cdot\frac{1+s\cdot C_O\cdot R_C}{\frac{R_O}{K_m\cdot R_i}+\frac{s\cdot C_O\cdot R_O\cdot R_C}{K_m\cdot R_i}+\left(1+\frac{R_L+R_S}{K_{mp}\cdot R_i}+\frac{s\cdot L}{K_{mp}\cdot R_i}\right)\cdot(1+s\cdot C_O\cdot(R_O+R_C))} \tag{A.45}$$

Expand the denominator and group like terms:

$$\frac{\hat{v}_O}{\hat{v}_C}=\frac{R_O}{R_i}\cdot\frac{1+s\cdot C_O\cdot R_C}{K_D+s\cdot C_O\cdot K_C+\left(\frac{s\cdot L}{K_{mp}\cdot R_i}\right)\cdot(1+s\cdot C_O\cdot(R_O+R_C))} \tag{A.46}$$

Where: $K_D=1+\frac{R_O}{K_m\cdot R_i}+\frac{R_L+R_S}{K_{mp}\cdot R_i}$ $\qquad K_C=(R_O+R_C)\cdot\left(1+\frac{R_L+R_S}{K_{mp}\cdot R_i}\right)+\frac{R_O\cdot R_C}{K_m\cdot R_i}$ (A.47)

By factoring this becomes:

$$\frac{\hat{v}_O}{\hat{v}_C}=\frac{R_O}{R_i}\cdot\frac{1+s\cdot C_O\cdot R_C}{K_D\cdot\left[1+s\cdot C_O\cdot\frac{K_C}{K_D}\right]\cdot\left[1+\left(\frac{s\cdot L}{K_{mp}\cdot R_i}\right)\cdot\left(\frac{1+s\cdot C_O\cdot(R_O+R_C)}{K_D+s\cdot C_O\cdot K_C}\right)\right]} \tag{A.48}$$

To simplify: $K_D\approx 1+\frac{R_O}{K_m\cdot R_i}$ $\qquad K_C\approx R_O$ $\qquad \frac{1+s\cdot C_O\cdot(R_O+R_C)}{K_D+s\cdot C_O\cdot K_C}\approx 1$ (A.49)

The simplified expression is:

$$\frac{\hat{v}_O}{\hat{v}_C}\approx\frac{R_O}{R_i}\cdot\frac{1+s\cdot C_O\cdot R_C}{\left(1+\frac{R_O}{K_m\cdot R_i}\right)\cdot\left(1+\frac{s\cdot C_O}{\frac{1}{R_O}+\frac{1}{K_m\cdot R_i}}\right)\cdot\left(1+\frac{s\cdot L}{K_{mp}\cdot R_i}\right)} \tag{A.50}$$

Continuous-Time Model
Derivation of Factored Pole/Zero Form

Starting with the control to output gain of equation A.22, expand Z_O and Z_L:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{Z_O}{\frac{Z_O}{K_m} + \frac{Z_L}{K'_{mp}} + R_i \cdot H_e(s)} = \frac{\frac{R_O \cdot (1 + s \cdot C_O \cdot R_C)}{1 + s \cdot C_O \cdot (R_O + R_C)}}{\frac{R_O \cdot (1 + s \cdot C_O \cdot R_C)}{K_m \cdot (1 + s \cdot C_O \cdot (R_O + R_C))} + \frac{s \cdot L + R_L + R_S}{K'_{mp}} + R_i \cdot H_e(s)} \tag{A.51}$$

Rationalize the numerator, factor R_O/R_i and expand He(s): (A.52)

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{R_O}{R_i} \cdot \frac{1 + s \cdot C_O \cdot R_C}{\frac{R_O}{K_m \cdot R_i} + \frac{s \cdot C_O \cdot R_O \cdot R_C}{K_m \cdot R_i} + \left(1 + \frac{R_L + R_S}{K'_{mp} \cdot R_i} + \frac{s \cdot L}{K'_{mp} \cdot R_i} + \frac{s}{\omega_n \cdot Q_z} + \frac{s^2}{\omega_n^{\,2}}\right) \cdot (1 + s \cdot C_O \cdot (R_O + R_C))}$$

Define:

$$\frac{s}{\omega_n \cdot Q} = \frac{s \cdot L}{K'_{mp} \cdot R_i} + \frac{s}{\omega_n \cdot Q_z} \tag{A.53}$$

Expand the denominator and group like terms:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{R_O}{R_i} \cdot \frac{1 + s \cdot C_O \cdot R_C}{K'_D + s \cdot C_O \cdot K'_C + \left(\frac{s}{\omega_n \cdot Q} + \frac{s^2}{\omega_n^{\,2}}\right) \cdot (1 + s \cdot C_O \cdot (R_O + R_C))} \tag{A.54}$$

Where: $K'_D = 1 + \frac{R_O}{K_m \cdot R_i} + \frac{R_L + R_S}{K'_{mp} \cdot R_i}$ $\quad K'_C = (R_O + R_C) \cdot \left(1 + \frac{R_L + R_S}{K'_{mp} \cdot R_i}\right) + \frac{R_O \cdot R_C}{K_m \cdot R_i}$ (A.55)

By factoring this becomes:

$$\frac{\hat{v}_O}{\hat{v}_C} = \frac{R_O}{R_i} \cdot \frac{1 + s \cdot C_O \cdot R_C}{K'_D \cdot \left[1 + s \cdot C_O \cdot \frac{K'_C}{K'_D}\right] \cdot \left[1 + \left(\frac{s}{\omega_n \cdot Q} + \frac{s^2}{\omega_n^{\,2}}\right) \cdot \left(\frac{1 + s \cdot C_O \cdot (R_O + R_C)}{K'_D + s \cdot C_O \cdot K'_C}\right)\right]} \tag{A.56}$$

To simplify: $K'_D \approx 1 + \frac{R_O}{K_m \cdot R_i}$ $\quad K'_C \approx R_O$ $\quad \frac{1 + s \cdot C_O \cdot (R_O + R_C)}{K'_D + s \cdot C_O \cdot K'_C} \approx 1$ (A.57)

The simplified expression is:

$$\frac{\hat{v}_O}{\hat{v}_C} \approx \frac{R_O}{R_i} \cdot \frac{1 + s \cdot C_O \cdot R_C}{\left(1 + \frac{R_O}{K_m \cdot R_i}\right) \cdot \left(1 + \frac{s \cdot C_O}{\frac{1}{R_O} + \frac{1}{K_m \cdot R_i}}\right) \cdot \left(1 + \frac{s}{\omega_n \cdot Q} + \frac{s^2}{\omega_n^{\,2}}\right)} \tag{A.58}$$

From equation A.53:

$$Q = \frac{1}{\pi \cdot \left(\frac{L}{K'_{mp} \cdot R_i \cdot T} - 0.5\right)} \tag{A.59}$$

Unified Linear Model Using General Gain Parameters

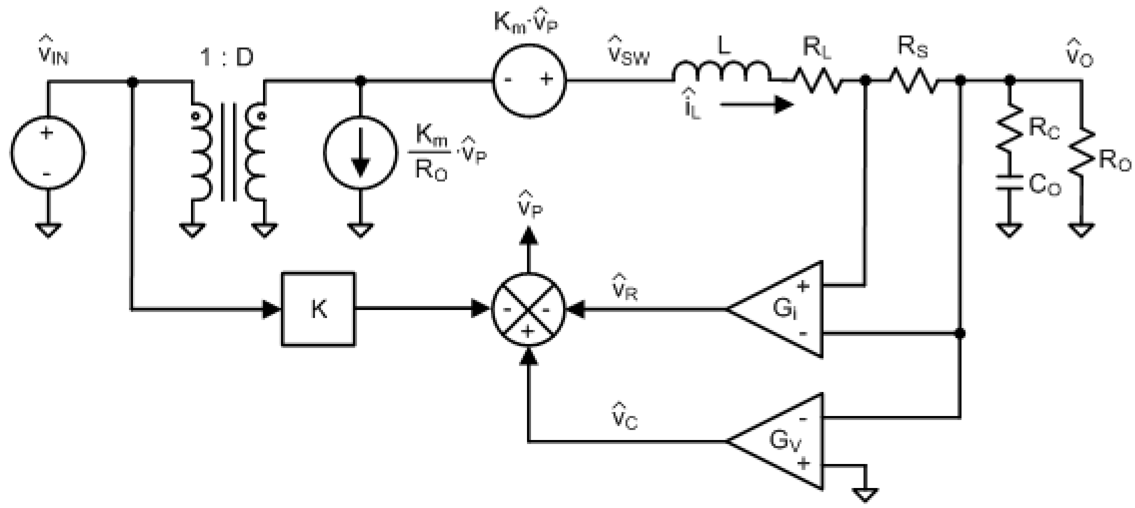

Figure A-4: Complete low frequency unified buck regulator linear model using general gain parameters. To model the current loop with sampling gain term, multiply K_m by $H_p(s)$ (valid for PCM1, VCM1 and VCM3 only). To model the control to output or voltage loop with sampling gain term, multiply G_i by H(s).

Derivation of model:

From equations A.4 and A.34:

$$\hat{v}_{SW} = \hat{v}_{IN} \cdot D + V_{IN} \cdot F_m(s) \cdot (\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K) \quad \text{(A.60)}$$

Let:

$$\hat{v}_P = \hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K \quad \text{(A.61)}$$

For the low frequency model let $V_{IN} \cdot F_m(s) = K_m$. Equation A.60 can now be expressed as:

$$\hat{v}_{SW} = \hat{v}_{IN} \cdot D + K_m \cdot \hat{v}_P \quad \text{(A.62)}$$

Comparing equation A.4 to equation A.62:

$$V_{IN} \cdot \hat{d} = K_m \cdot \hat{v}_P \qquad \text{therefore:} \qquad \hat{d} = \frac{K_m}{V_{IN}} \cdot \hat{v}_P \quad \text{(A.63)}$$

The power stage of figure A-1 (a) shows the relationship of the input current to the inductor current:

$$\hat{i}_{IN} = \hat{i}_L \cdot D + I_L \cdot \hat{d} \quad \text{(A.64)}$$

Since $I_L = V_O/R_O$ and $V_O = V_{IN} \cdot D$, substitution of equation A.63 yields:

$$\hat{i}_{IN} = D \cdot \left(\hat{i}_L + \frac{K_m}{R_O} \cdot \hat{v}_P \right) \quad \text{(A.65)}$$

K can be found from the general gain parameters using equation A.42. Substitution of variables into equations A.10 and A.13 leads to an alternate form using averaged model parameters:

$$K = K_I - D \cdot K_O \tag{A.66}$$

TABLE A-1
DERIVATION OF K USING AVERAGED MODEL PARAMETERS

Mode	K_I	$-D \cdot K_O$	K
PCM1	$0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$	$-D \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)$	$0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D'$
PCM2	$0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$	$-D \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} - K_{SL}\right) \cdot D$	$0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D' + K_{SL} \cdot D^2$
VCM1	0	$-D \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'\right)$	$-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D'$
VCM2	$-K_{SL} \cdot D'$	$-D \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} - K_{SL}\right) \cdot D'$	$-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D' - K_{SL} \cdot (D')^2$
EPCM1	$-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D$	$-D \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)$	$-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D'$
EPCM2	$\left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$	$-D \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)$	$-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D' + K_{SL} \cdot D$
EVCM1	0	$-D \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'\right)$	$0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D'$
EVCM2	$-K_{SL} \cdot D'$	$-D \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'\right)$	$0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D' - K_{SL} \cdot D'$
VCM3	$-K_{SL} \cdot D'$	$-D \cdot \left(0.5 \cdot R_i \cdot \frac{T}{L} \cdot D'\right)$	$-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D' - K_{SL} \cdot D'$
EPCM3	$\left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$	$-D \cdot \left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$	$\left(-0.5 \cdot R_i \cdot \frac{T}{L} + K_{SL}\right) \cdot D \cdot D'$
EPCM4	$\left(K_{SL} - 0.5 \cdot R_i \cdot \frac{T}{L}\right) \cdot D$	$-D \cdot \left(-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D\right)$	$-0.5 \cdot R_i \cdot \frac{T}{L} \cdot D \cdot D' + K_{SL} \cdot D$

The low frequency linear model of figure A-4 can be used for all transfer functions including the input impedance and output impedance. The line to output and control to output transfer functions can be reduced to the simplified transfer functions of the averaged model represented by equations 39 and 40. The only difference is the inductor pole frequency which becomes:

$$\omega_L = \frac{K_m \cdot R_i}{L} \tag{A.67}$$

This is considered to be the “true” inductor pole frequency without the sampling gain term.

Derivation of $H_P(s)$ from H(s)

Let the control to inductor current transfer function of the continuous-time model from equation A.32 equal that of the unified model from equation A.40.

$$\frac{\hat{i}_L}{\hat{v}_C} = \frac{K_m}{Z_O + Z_L + K_m \cdot R_i \cdot H(s)} = \frac{K_m \cdot H_P(s)}{Z_O + Z_L + K_m \cdot R_i \cdot H_P(s)} \tag{A.68}$$

Solve for $H_P(s)$.

$$H_P(s) = \frac{Z_O + Z_L}{Z_O + Z_L + K_m \cdot R_i \cdot (H(s) - 1)} \tag{A.69}$$

Evaluation of H(s)-1 yields:

$$H_P(s) = \frac{Z_O + Z_L}{Z_O + Z_L + K_m \cdot R_i \cdot \left[\frac{Z_L}{R_i} \cdot \left(\frac{1}{K'_{mp}} - \frac{1}{K_m}\right) + \frac{s}{\omega_n \cdot Q_z} + \frac{s^2}{{\omega_n}^2}\right]} \tag{A.70}$$

Utilization of equation A.70 in the unified model does not result in accurate plots of the current loop gain for all operating modes. It does yield accurate plots of the control to output gain.

To simplify the analysis, let $Z_L/R_i = s \cdot L/R_i$. Evaluation of ω_n and Q_Z results in:

$$H_P(s) = \frac{1 + \frac{Z_O}{Z_L}}{1 + \frac{Z_O}{Z_L} + \frac{K_m \cdot R_i}{Z_L} \cdot \left[s \cdot \left(\frac{L}{R_i} \cdot \left(\frac{1}{K'_{mp}} - \frac{1}{K_m}\right) - \frac{T}{2}\right) + \frac{s^2}{{\omega_n}^2}\right]} \tag{A.71}$$

Let:

$$K_e = \frac{L}{R_i} \cdot \left(\frac{1}{K'_{mp}} - \frac{1}{K_m}\right) - \frac{T}{2} \tag{A.72}$$

This allows $H_P(s)$ to be expressed as:

$$H_P(s) = \frac{1 + \frac{Z_O}{Z_L}}{1 + \frac{Z_O}{Z_L} + \frac{K_m \cdot R_i}{Z_L} \cdot \left(s \cdot K_e + \frac{s^2}{{\omega_n}^2}\right)} \tag{A.73}$$

K_e is evaluated for each operating mode in table A-2.

TABLE A-2
EVALUATION OF K_e FOR EACH OPERATING MODE

Mode	$\frac{1}{K'_{mp}}$	$\frac{1}{K_m}$	K_e
PCM1	$R_i \cdot \frac{T}{L} \cdot (1-D) + \frac{V_{SL}}{V_{IN}}$	$(0.5-D) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}$	0
PCM2	$R_i \cdot \frac{T}{L} \cdot (1-D) + K_{SL} \cdot D$	$(0.5-D) \cdot R_i \cdot \frac{T}{L} + 2 \cdot K_{SL} \cdot D$	$-K_{SL} \cdot D \cdot \frac{L}{R_i}$
VCM1	$R_i \cdot \frac{T}{L} \cdot D + \frac{V_{SL}}{V_{IN}}$	$(D-0.5) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}$	0
VCM2	$R_i \cdot \frac{T}{L} \cdot D + K_{SL} \cdot (1-D)$	$(D-0.5) \cdot R_i \cdot \frac{T}{L} + 2 \cdot K_{SL} \cdot (1-D)$	$-K_{SL} \cdot D' \cdot \frac{L}{R_i}$
EPCM1	$\frac{V_{SL}}{V_{IN}}$	$(D-0.5) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}$	$-D \cdot T$
EPCM2	K_{SL}	$(D-0.5) \cdot R_i \cdot \frac{T}{L} + K_{SL}$	$-D \cdot T$
EVCM1	$\frac{V_{SL}}{V_{IN}}$	$(0.5-D) \cdot R_i \cdot \frac{T}{L} + \frac{V_{SL}}{V_{IN}}$	$-D' \cdot T$
EVCM2	K_{SL}	$(0.5-D) \cdot R_i \cdot \frac{T}{L} + K_{SL}$	$-D' \cdot T$
VCM3	$R_i \cdot \frac{T}{L} \cdot D + K_{SL}$	$(D-0.5) \cdot R_i \cdot \frac{T}{L} + K_{SL}$	0
EPCM3	$K_{SL} \cdot (1-D) + \frac{V_{SL}}{V_{IN}}$	$(D-0.5) \cdot R_i \cdot \frac{T}{L} + (1-2 \cdot D) \cdot K_{SL} + \frac{V_{SL}}{V_{IN}}$	$-D \cdot T + K_{SL} \cdot D \cdot \frac{L}{R_i}$
EPCM4	$K_{SL} + \frac{V_{SL}}{V_{IN}}$	$(D-0.5) \cdot R_i \cdot \frac{T}{L} + K_{SL} + \frac{V_{SL}}{V_{IN}}$	$-D \cdot T$

K_e=0 for PCM1, VCM1 and VCM3.

Evaluation of the high frequency asymptote is done by recognizing that $1+\frac{Z_O}{Z_L} \approx 1$.

With Z_L=s·L, $H_P(s)$ reduces to:

$$H_P(s) = \frac{1}{1 + s \cdot \frac{Q}{\omega_n}} \qquad \text{where} \qquad Q = \frac{1}{\pi \cdot \left(\frac{L}{K_m \cdot R_i \cdot T} \right)} \tag{A.74}$$

For PCM1, VCM1 and VCM3, this value of Q is equivalent to that defined by equation A.59.

Unified Linear Model Using General Gain Parameters
Low Frequency Model of Figure A-4
Line Rejection

Starting from equation A.37 with $H_P(s)=1$:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + K_m \cdot \left(\hat{v}_C - \hat{v}_O \cdot \frac{R_i}{Z_O} - \hat{v}_{IN} \cdot K \right) \right] \cdot \frac{Z_O}{Z_O + Z_L} \tag{A.75}$$

Since $\hat{v}_C = -\hat{v}_O \cdot G_V$:

$$\hat{v}_O = \left[\hat{v}_{IN} \cdot D + K_m \cdot \left(-\hat{v}_O \cdot G_V - \hat{v}_O \cdot \frac{R_i}{Z_O} - \hat{v}_{IN} \cdot K \right) \right] \cdot \frac{Z_O}{Z_O + Z_L} \tag{A.76}$$

This allows the closed loop low frequency line rejection to be expressed as:

$$\frac{\hat{v}_O}{\hat{v}_{IN}} = \frac{(D - K \cdot K_m) \cdot Z_O}{Z_O + Z_L + K_m \cdot G_V \cdot Z_O + K_m \cdot R_i} \tag{A.77}$$

By setting $G_V=0$, this reduces to the open loop low frequency audio susceptibility equation.

Note: Introduction of either $H_P(s)$ or $H(s)$ will still result in a deviation from the actual response at higher frequency.

Unified Linear Model Using General Gain Parameters
Low Frequency Model of Figure A-4
Output Impedance

Starting with $\hat{v}_{SW}$, write the transfer functions from the model of figure A-4. There is no perturbation from the input, so $\hat{v}_{IN} = 0$.

$$\hat{v}_{SW} = K_m \cdot (-\hat{v}_O \cdot G_V - \hat{i}_L \cdot R_i) \tag{A.78}$$

The output voltage is perturbed by an external current source such that:

$$\hat{v}_O = (\hat{i}_L + \hat{i}_O) \cdot Z_O \tag{A.79}$$

The inductor current can be expressed as:

$$\hat{i}_L = \frac{(\hat{v}_{SW} - \hat{v}_O)}{Z_L} \tag{A.80}$$

Combining equations, the closed loop output impedance is found to be:

$$\frac{\hat{v}_O}{\hat{i}_O} = \frac{Z_O}{1 + \dfrac{Z_O \cdot (1 + K_m \cdot G_V)}{Z_L + K_m \cdot R_i}} \tag{A.81}$$

By setting $G_V=0$, this reduces to the open loop output impedance.

To incorporate the sampling gain term, multiply R_i by $H(s)$.

Unified Linear Model Using General Gain Parameters
Low Frequency Model of Figure A-4
Input Impedance

From A.60, write the equation for $\hat{v}_{SW}$:

$$\hat{v}_{SW} = \hat{v}_{IN} \cdot D + K_m \cdot (\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K) = \hat{i}_L \cdot (Z_O + Z_L) \quad \text{(A.82)}$$

Write the equation for $\hat{i}_{IN}$ from A.61 and A.65:

$$\hat{i}_{IN} = \hat{i}_L \cdot D + \frac{K_m \cdot D}{R_O} \cdot (\hat{v}_C - \hat{i}_L \cdot R_i - \hat{v}_{IN} \cdot K) \quad \text{(A.83)}$$

From the model of figure A-4:

$$\hat{v}_C = -\hat{v}_O \cdot G_V = -\hat{i}_L \cdot G_V \cdot Z_O \quad \text{(A.84)}$$

Combining equations A.82 and A.84:

$$\hat{i}_L = \frac{\hat{v}_{IN} \cdot (D - K \cdot K_m)}{Z_O + Z_L + K_m \cdot G_V \cdot Z_O + K_m \cdot R_i} \quad \text{(A.85)}$$

Combining equations A.83 and A.84:

$$\hat{i}_L = \frac{\hat{i}_{IN} + \hat{v}_{IN} \cdot \frac{K \cdot K_m \cdot D}{R_O}}{D - \frac{K_m \cdot G_V \cdot Z_O \cdot D}{R_O} - \frac{K_m \cdot R_i \cdot D}{R_O}} \quad \text{(A.86)}$$

Set equation A.85 equal to A.86. After much algebraic manipulation, the closed loop input impedance is found to be:

$$\frac{\hat{v}_{IN}}{\hat{i}_{IN}} = -\frac{R_O}{D^2} \cdot \frac{1 + \frac{Z_O + Z_L}{K_m \cdot G_V \cdot Z_O + K_m \cdot R_i}}{1 + \frac{\frac{K \cdot K_m}{D} \cdot (R_O + Z_O + Z_L) - R_O}{K_m \cdot G_V \cdot Z_O + K_m \cdot R_i}} \quad \text{(A.87)}$$

By setting G_V=0, the open loop input impedance is found.

Note: Introduction of either $H_P(s)$ or H(s) will still result in a deviation from the actual response at higher frequency.

LM3495 Unified Linear Model using General Gain Parameters
Low Frequency Model without Sampling Gain Term

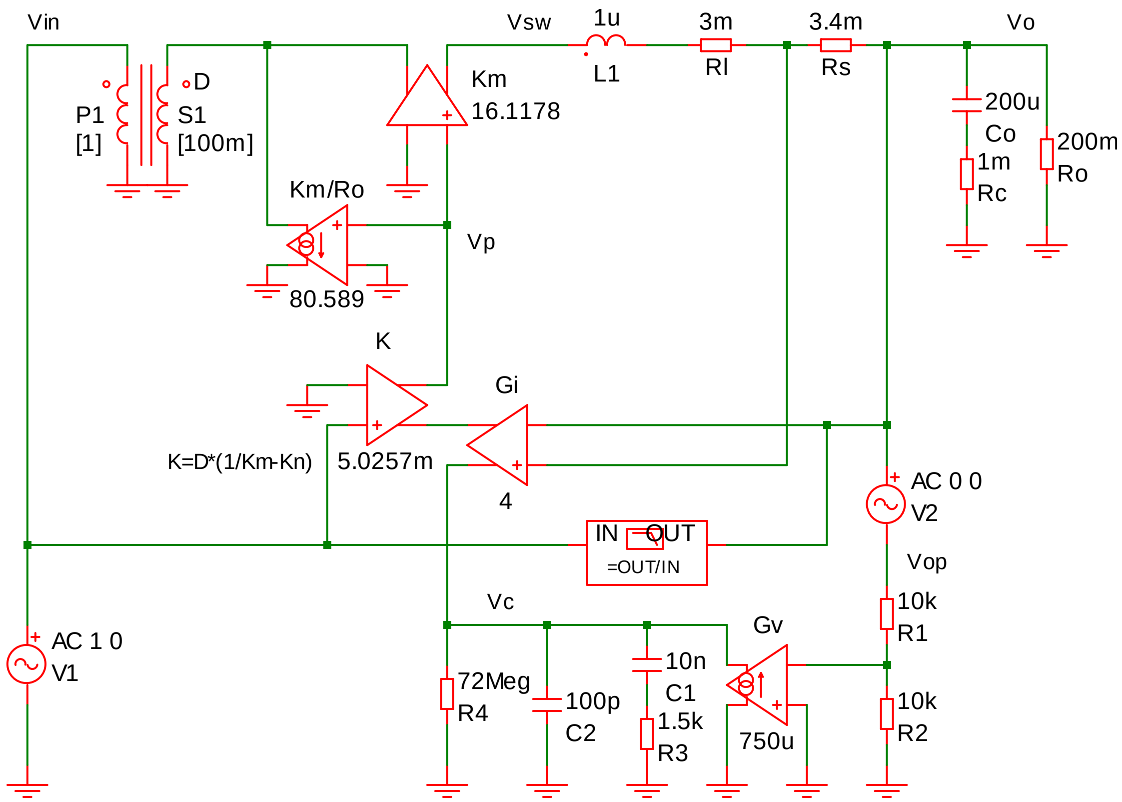

Figure B-1: SIMETRIX LM3495 Linear SPICE Model

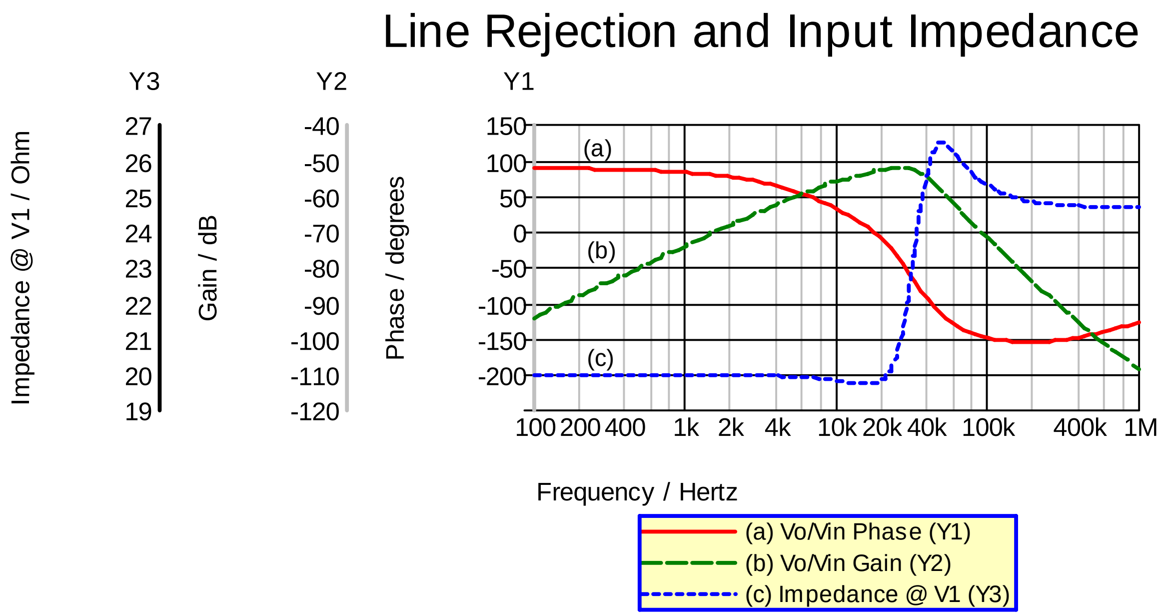

Figure B-2: Line Rejection and Negative Input Impedance

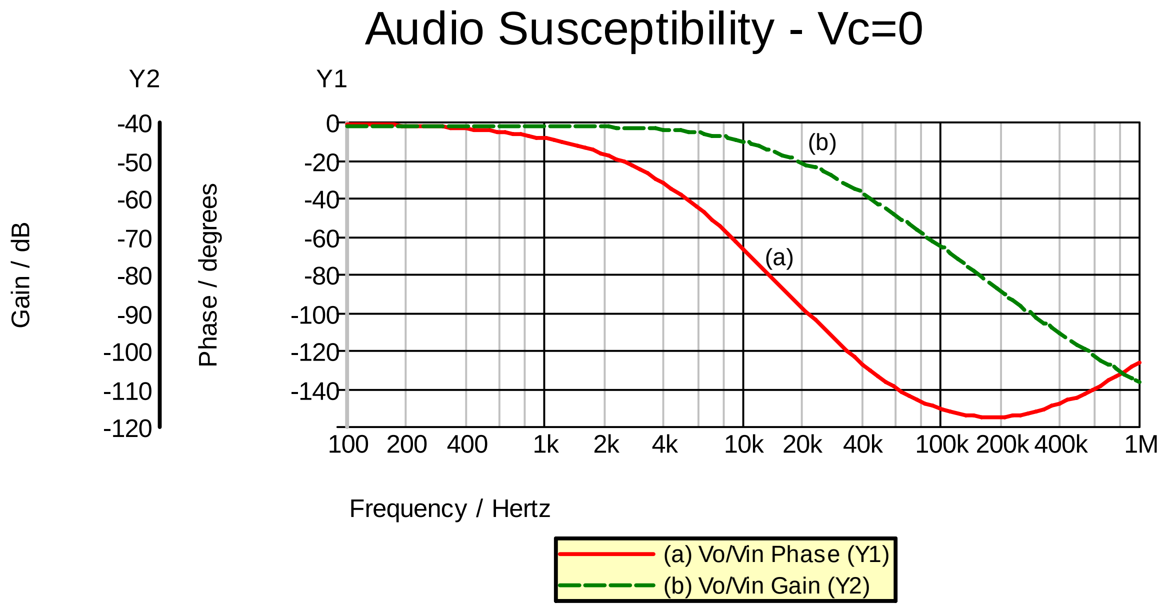

Figure B-3: Audio Susceptibility

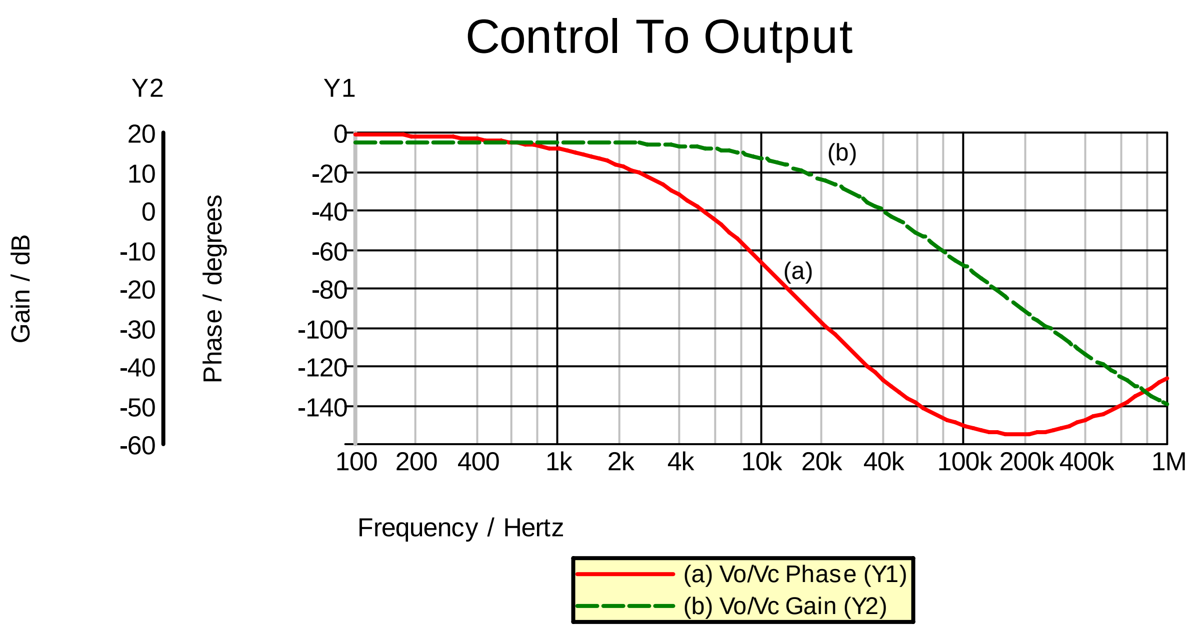

Figure B-4: Control to Output

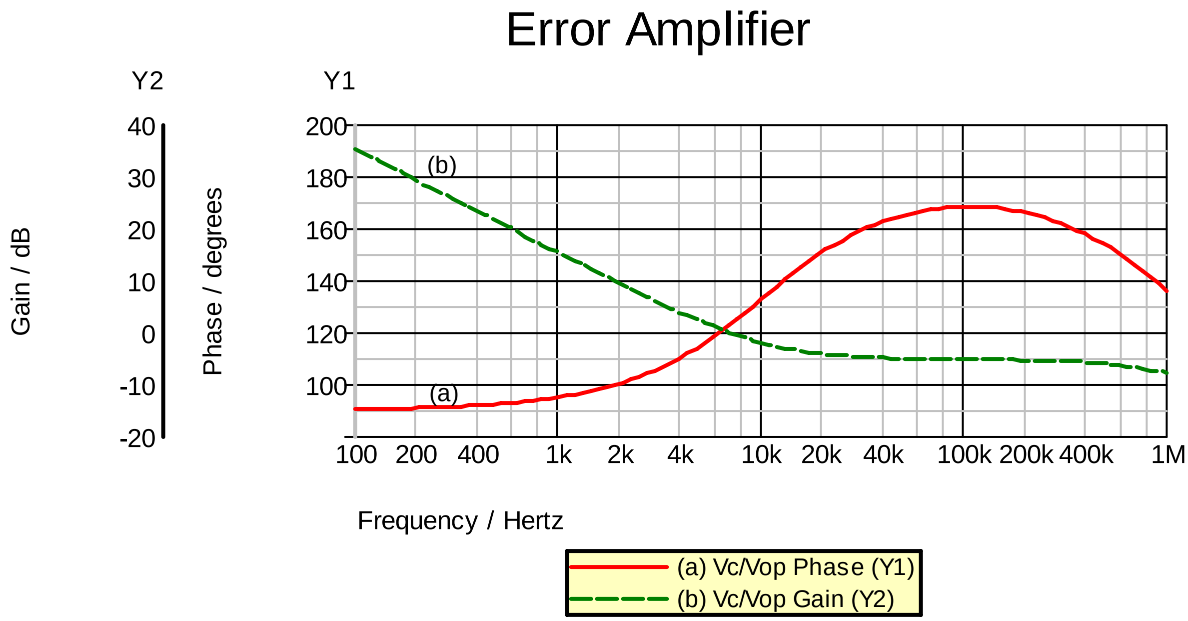

Figure B-5: Error Amplifier

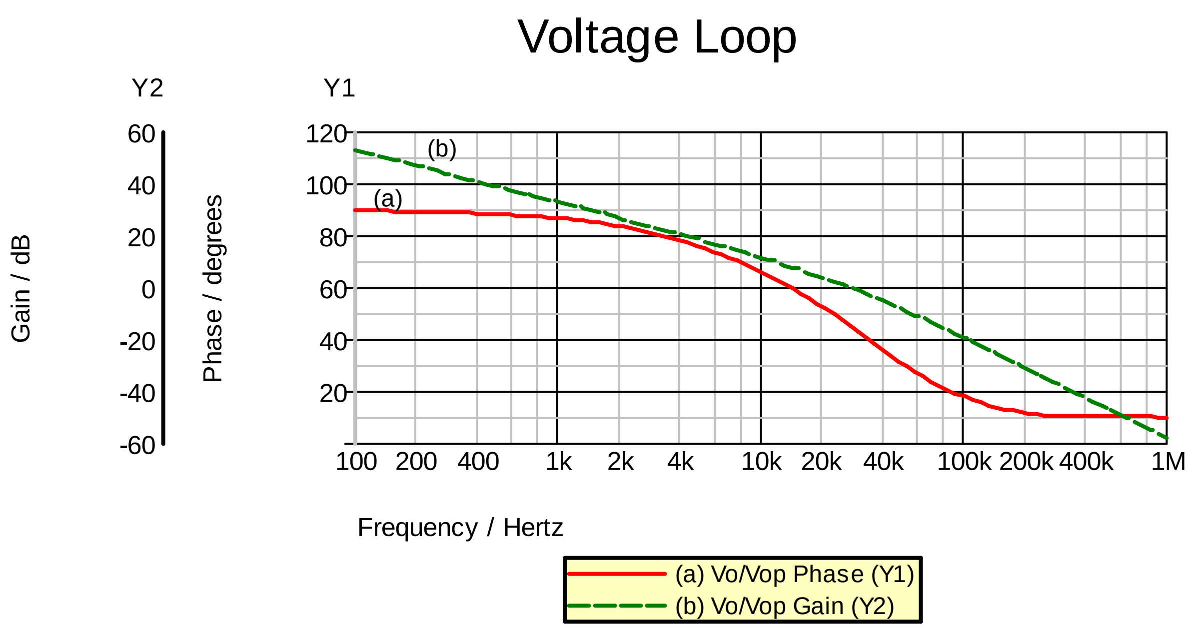

Figure B-6: Voltage Loop

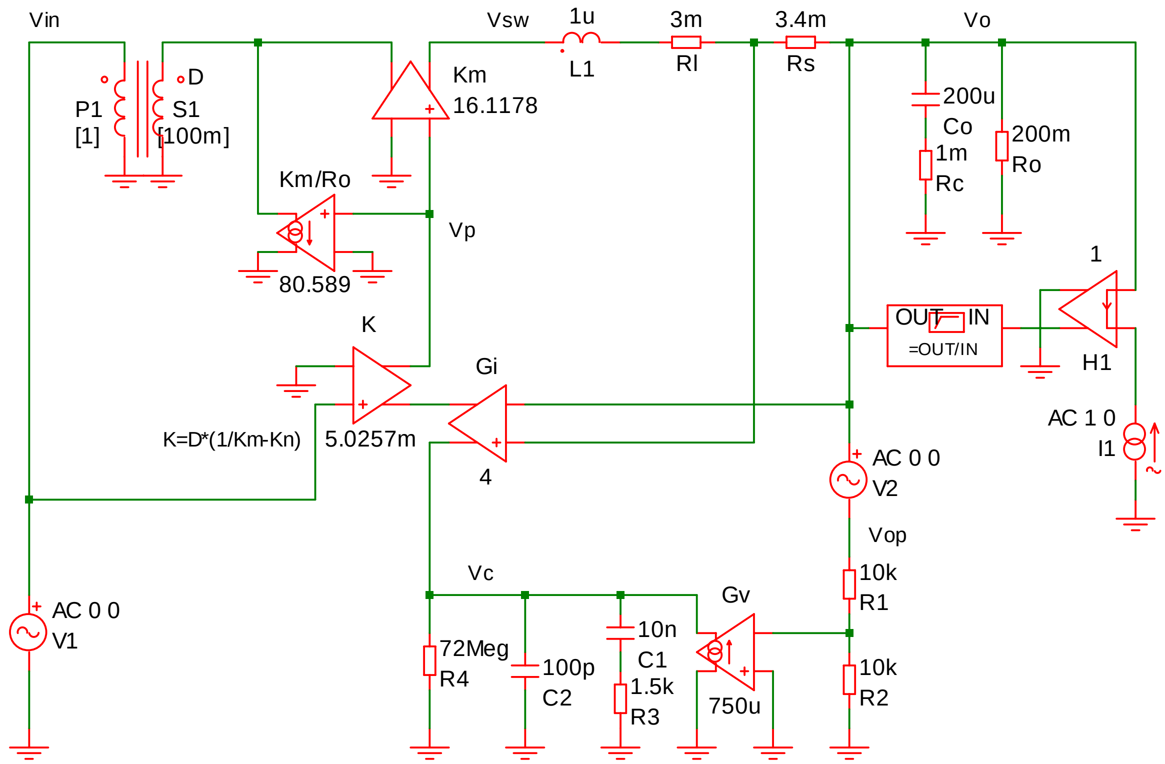

Figure B-7: SIMETRIX LM3495 Linear Model for Output Impedance

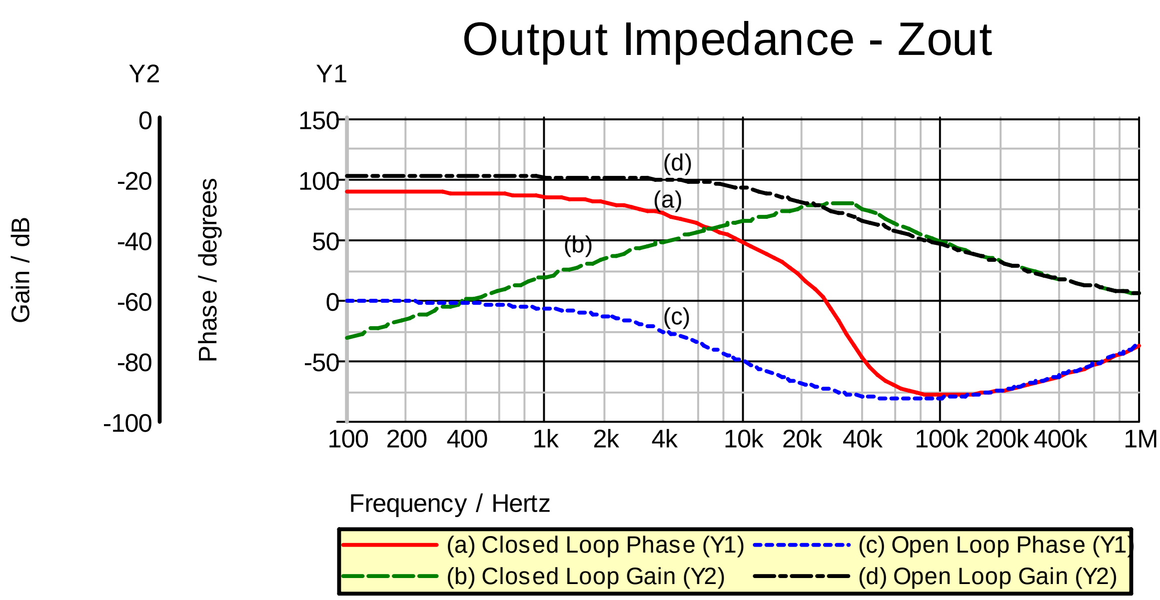

Figure B-8: Output Impedance

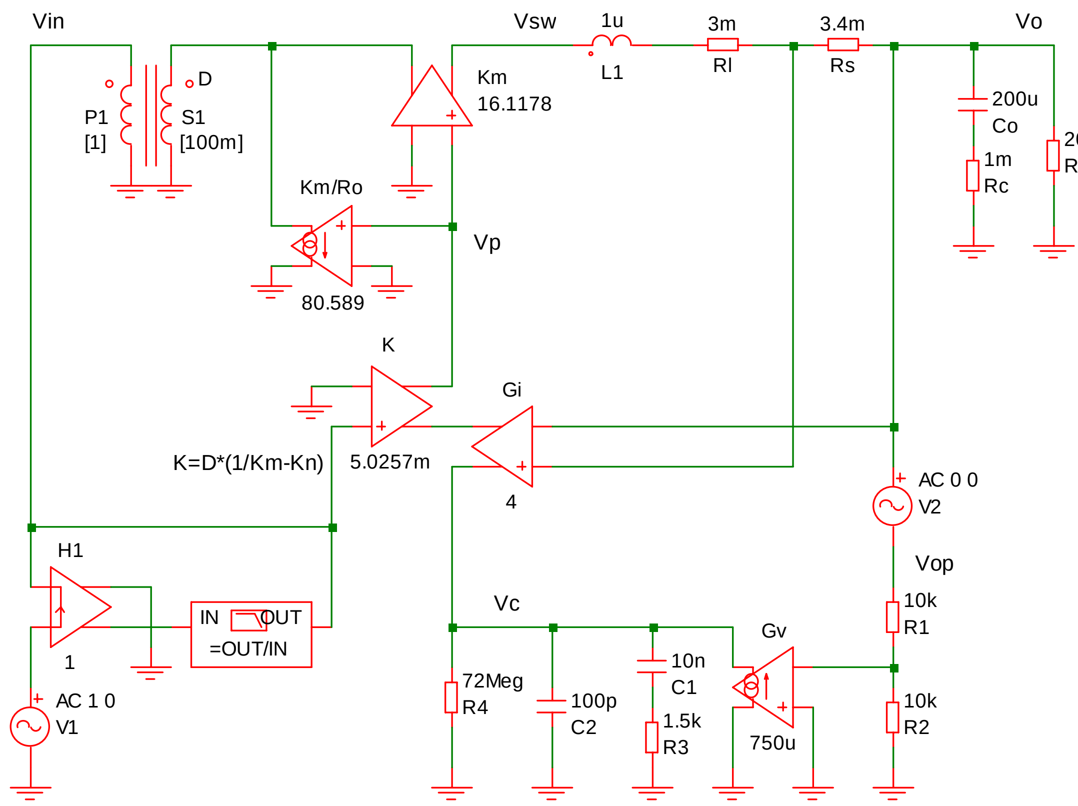

Figure B-9: SIMETRIX LM3495 Linear Model for Input Impedance

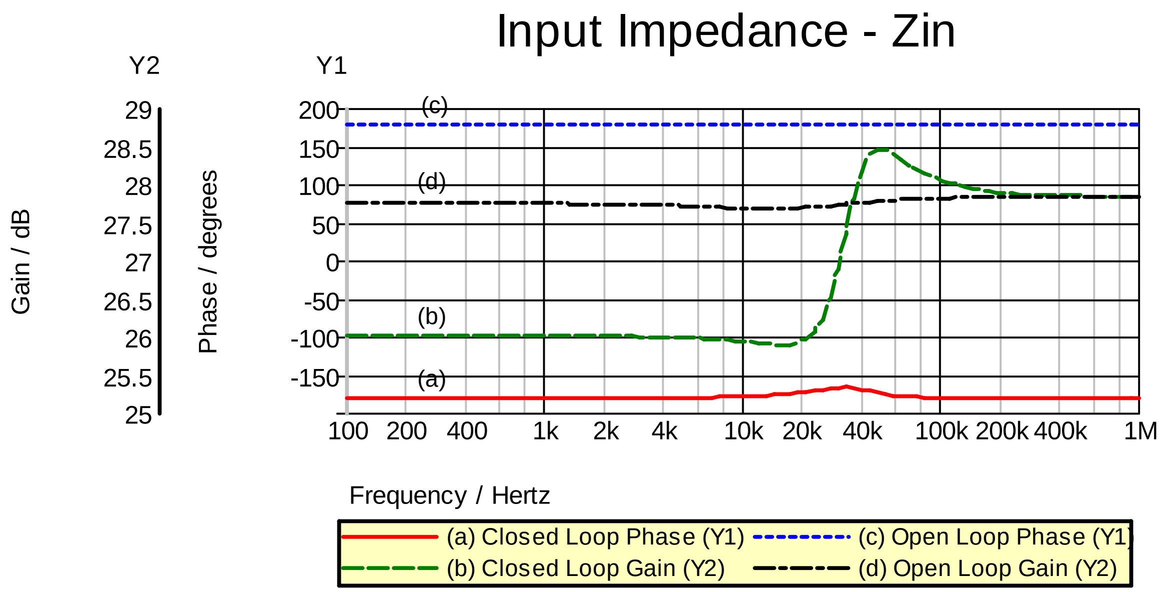

Figure B-10: Input Impedance

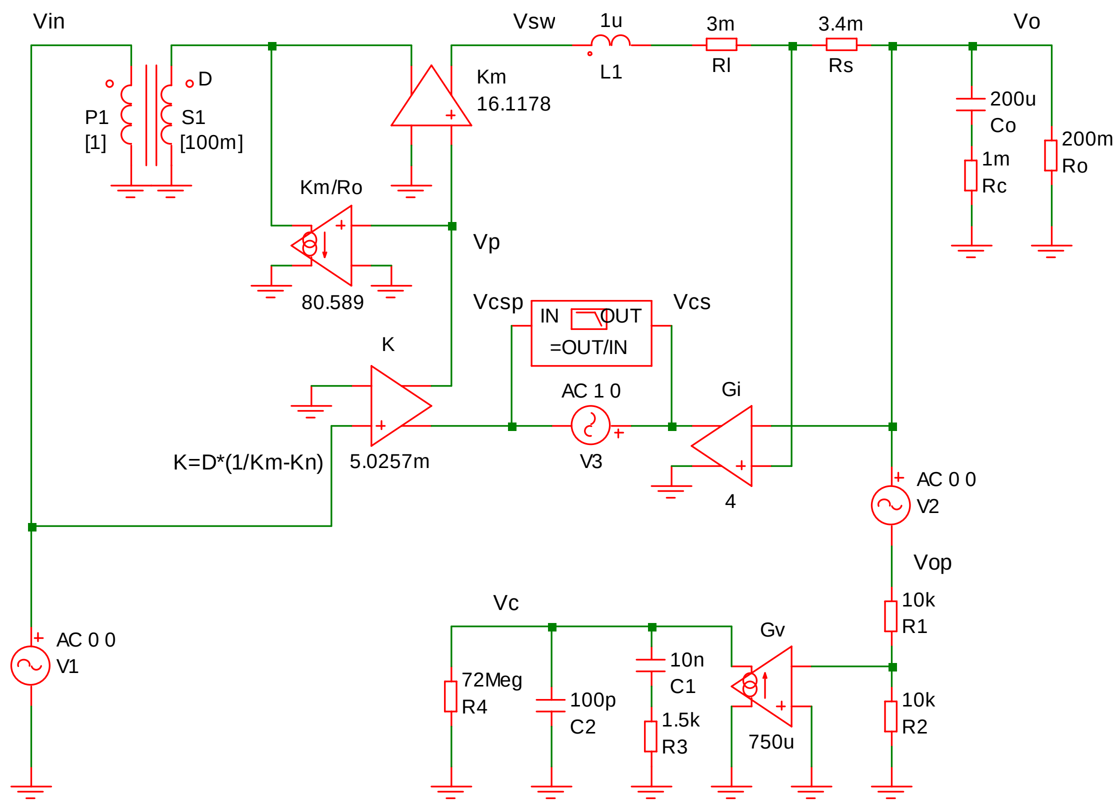

Figure B-11: SIMETRIX LM3495 Linear Model for Current Loop

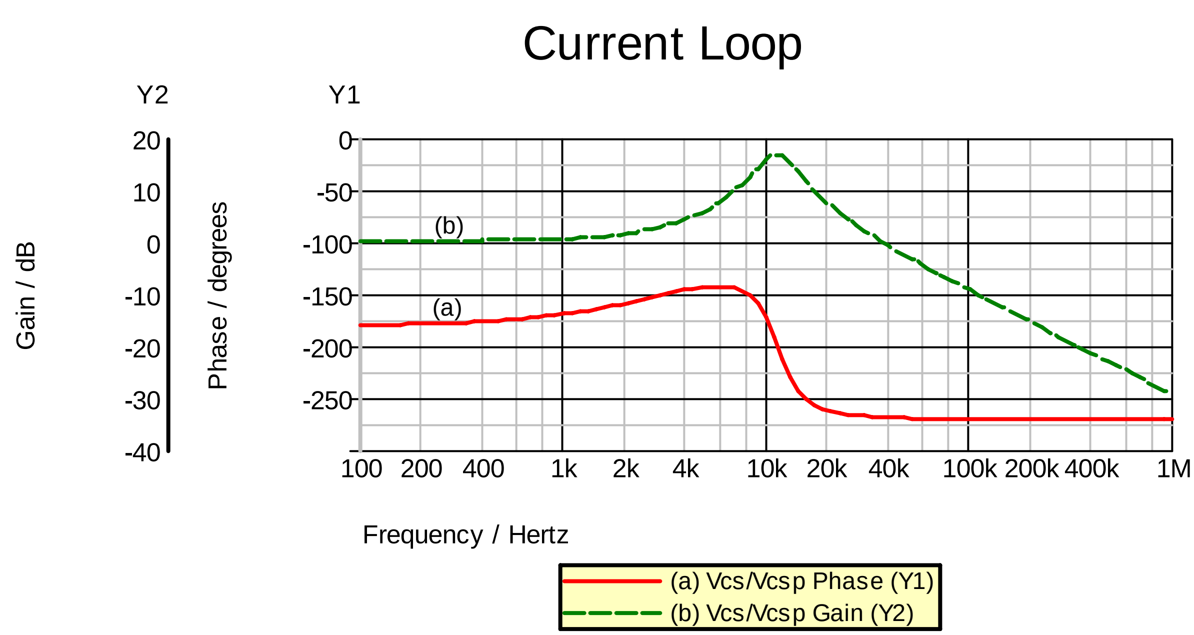

Figure B-12: Current Loop

EMULATED CURRENT MODE CONTROL FOR BUCK REGULATORS USING SAMPLE AND HOLD TECHNIQUE

Small Signal Linear Analysis and Comparison to Peak and Valley Methods

by

Robert Sheehan
Principal Applications Engineer
National Semiconductor Corporation
Santa Clara, CA

ERRATA

For

PES02
Tuesday, October 24, 2006
8:30am – 9:30am

Power Electronics Technology Exhibition and Conference
October 24-26, 2006
Long Beach Convention Center
Long Beach, CA

Corrections to the Conference Proceedings CD version, which are included in the print version:

1. Figure 4: Delete last sentence, "By setting H(s)=1, the averaged model is obtained."

2. Table 2A: VCM1, change $S_n=(V_{IN}-V_O)\cdot R_i/L$ to $S_n=V_O\cdot R_i/L$.

3. Table 5: Last section for PCM2 and VCM2, change K_{SL} from "0.05" to "0.10" (two places).

4. Figure 10: Graph legends, change last entries for "Gain Q=0.318" to "Phase Q=0.318" (three places).

5. Equation 71: Change from "$i_L\cdot R_i$ - 0.5…" to "$i_L\cdot R_i$ + 0.5…"

6. Section 4. General Slope Compensation Requirements, second paragraph: Change from "when the sum of the inductor current's slope…" to "when the sum of the sensed inductor current's slope…"

Additional sections which are not on the Conference Proceedings CD version:

Appendix A

Appendix B

Notes: